Wavelets in NMR-imaging

Plan

- Wavelet theory:

From Fourier to wavelet analysis

Wavelet decomposition

Multiresolution analysis

- Applications

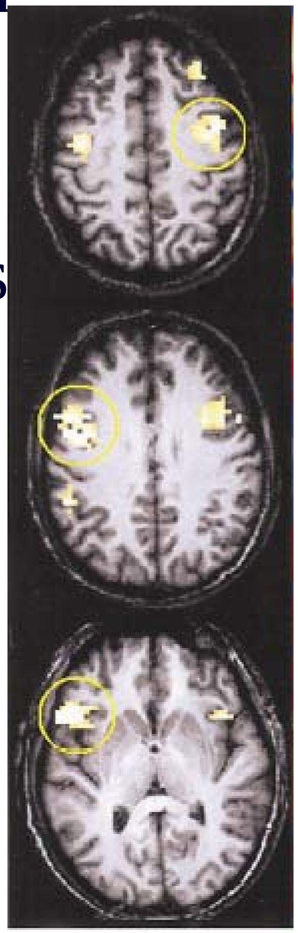
Signal denoising

Wavelet MRI encoding

Magnetic Resonance Imaging

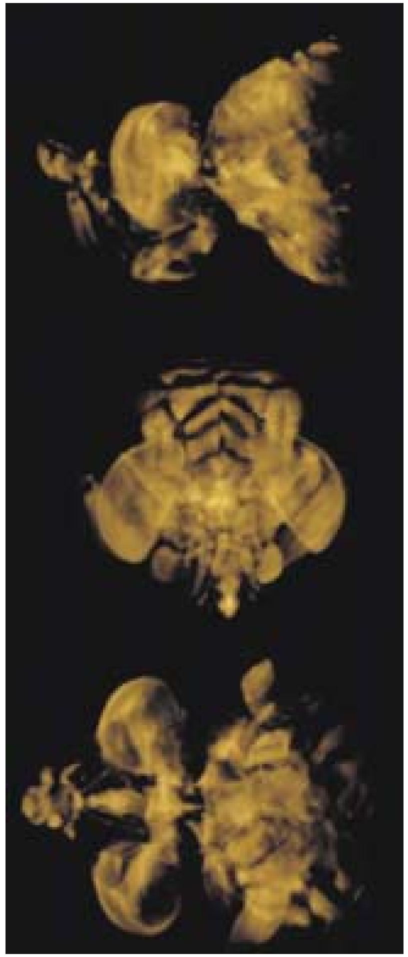
- Functional MRI

Used for detection of activation of brain regions as a reaction to different kinds of tasks.



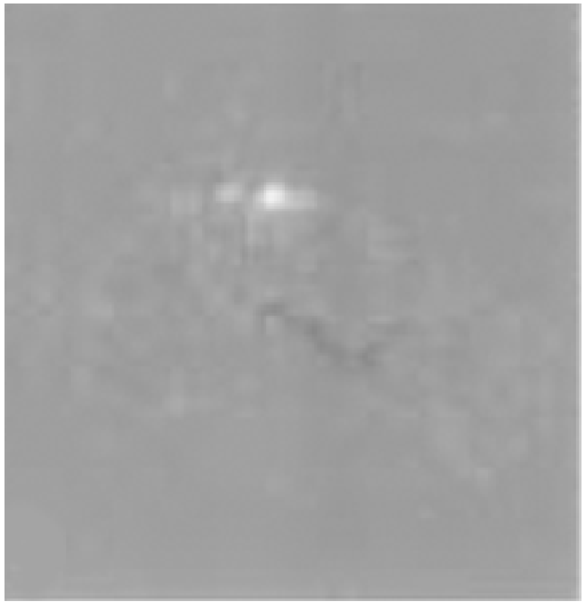
- MRI

Used for non-invasive observation of humans, animals, plants, insects and materials.

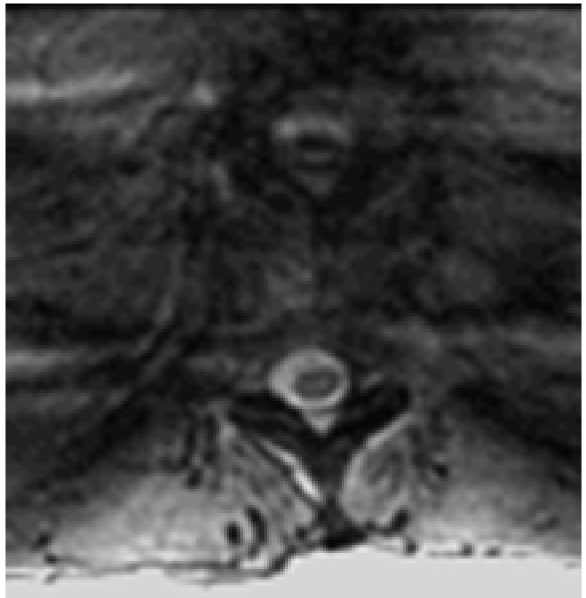


Problems in MRI

- Low signal to noise ratio in fMRI images



- Effects, rising due to long scanning time, e.g. subject movement



Wavelets

- From Fourier to wavelet analysis
- Wavelet decomposition
- Multiresolution analysis

From Fourier analysis

- Fourier series

$$f(x) = a_0 + \sum_{1}^{\infty} (a_k \cos(kx) + b_k \sin(kx))$$

- Fourier coefficients

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx$$

$$a_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(kx) dx$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin(kx) dx$$

Haar decomposition

- Haar basis

$$h(x) = \left[\begin{array}{r} 1, x \in [0, 1/2) \\ -1, x \in [1/2, 1) \end{array} \right.$$

- Haar series

$$h_0 = 1$$

$$h_{j,k} = 2^{\frac{j}{2}} h(2^j x - k)$$

$$f(x) = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} \langle h_{j,k}(x) f(x) \rangle h_{j,k}$$

Wavelet analysis

- Wavelet basis

$$\int_0^\infty |\hat{\psi}(t\xi)| \frac{dt}{t} = 1$$

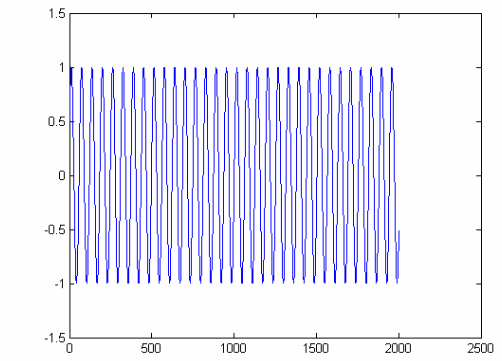
- Shifts and scaling

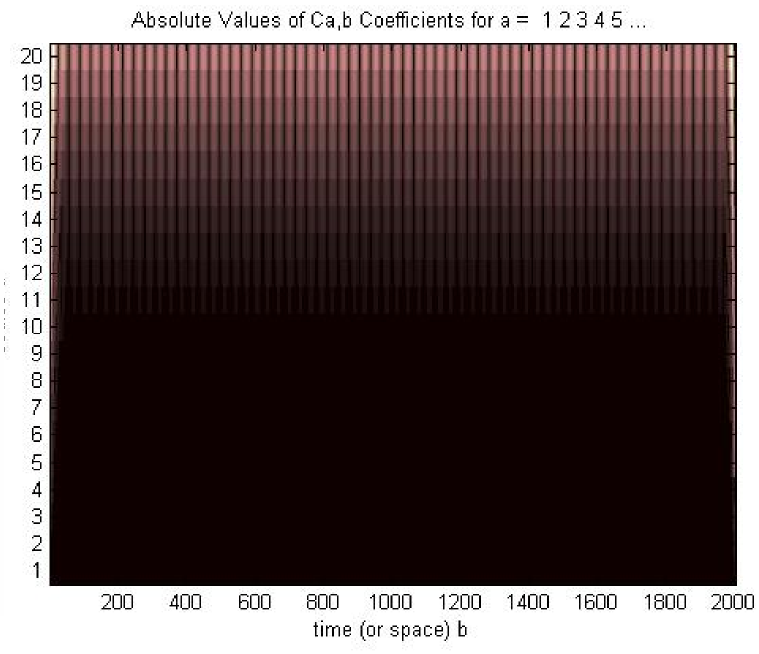
$$\psi_{j,k}(x) = 2^{\frac{j}{2}} \psi(2^j x - k)$$

- Coefficients and series

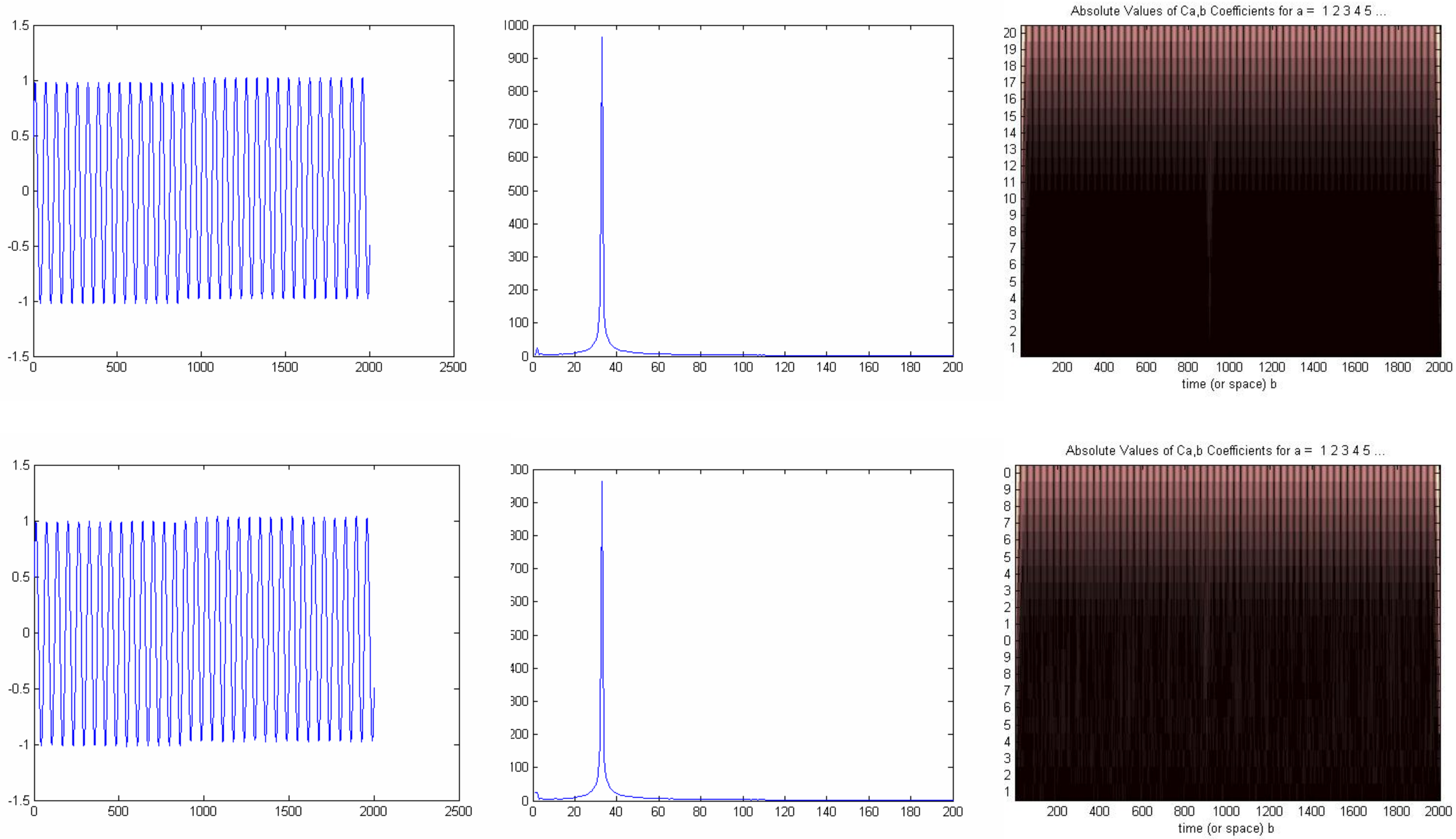
$$C_{j,k} = \langle f, \psi_{j,k} \rangle = 2^{\frac{j}{2}} \int f(x) \overline{\psi(2^j x - k)} dx$$

$$f(x) = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} C_{j,k} \psi_{j,k}(x)$$



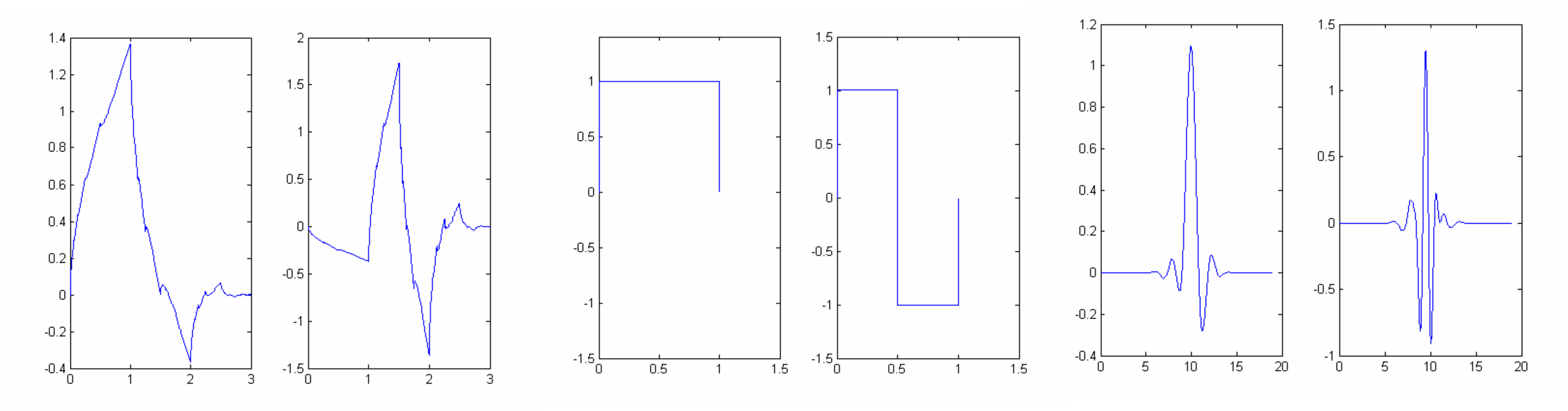


Wavelet and Fourier representations



Multiresolution analysis

- Scaling and wavelet functions



- Series and coefficients

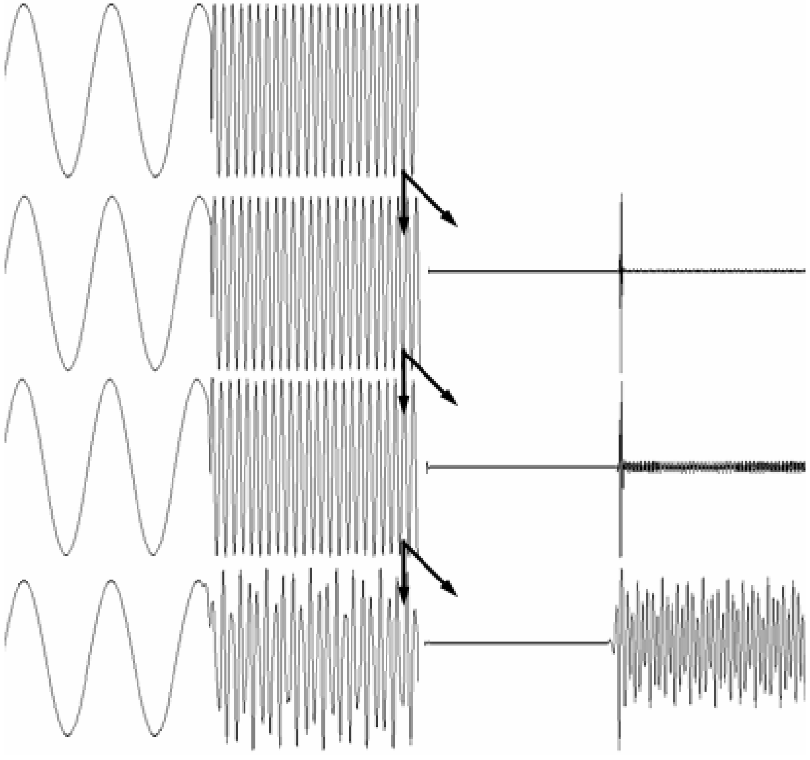
$$f(x) = \sum_{k} A_{j_n,k}\varphi_{j_n,k} + \sum_{j \geq j_n,k} C_{j,k}\psi_{j,k}$$

$$A_{j,k} = 2^{\frac{j}{2}} \int f(x)\varphi(2^j x - k)dx$$

$$C_{j,k} = 2^{\frac{j}{2}} \int f(x)\psi(2^j x - k)dx$$

1D and 2D multiresolution analysis

- 1D



- 2D

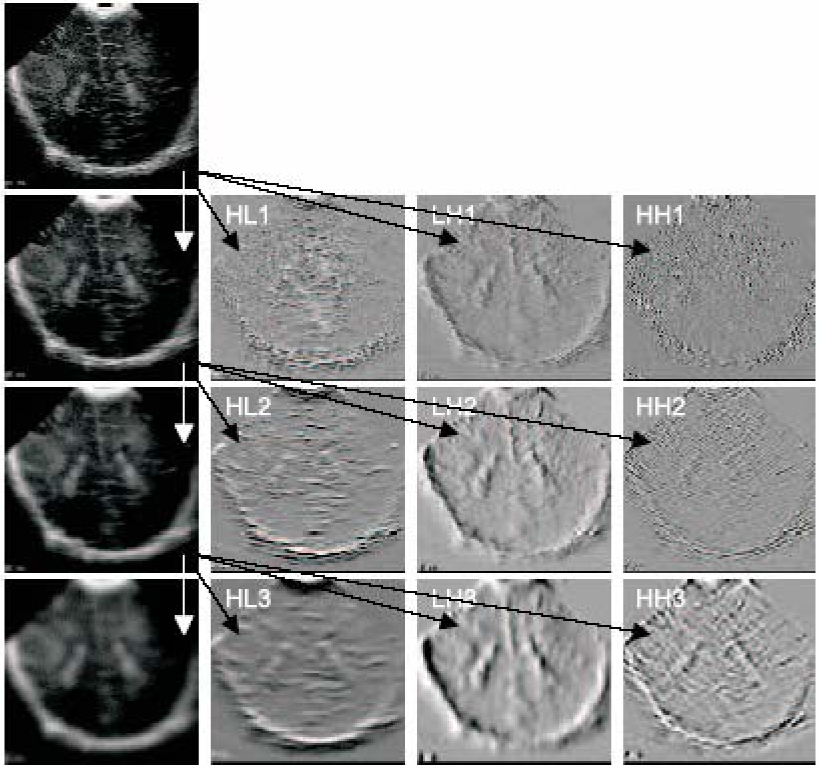


Image denoising

- Simple thresholding
- Using probability of important detail presence

Noise suppression

- Threshold parameter

$$\lambda = \frac{1}{n_{pix}} \sum \Sigma^2(n)$$

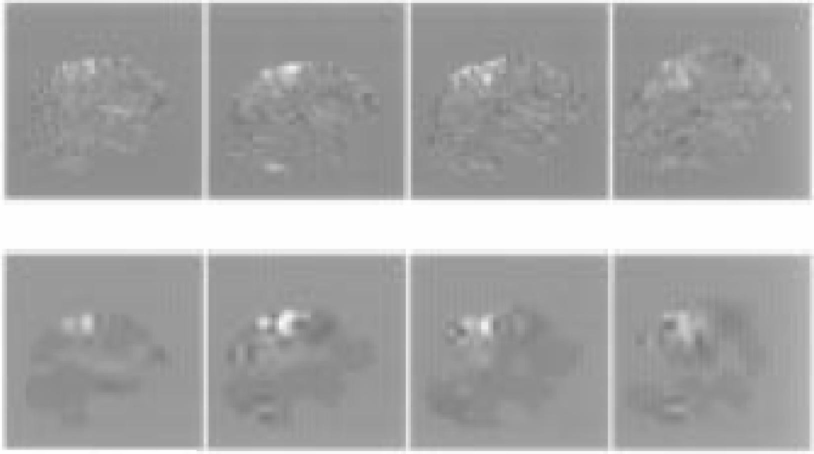
- Soft and hard thresholding

$$C_{j,k} = \begin{cases} 0, |C_{j,k}| < \lambda \\ C_{j,k}, |C_{j,k}| > \lambda \end{cases}$$

$$C_{j,k} = \begin{cases} C_{j,k} - \lambda, C_{j,k} > \lambda \\ 0, |C_{j,k}| \le \lambda \\ C_{j,k} + \lambda, C_{j,k} < \lambda \end{cases}$$

Examples

- Hard thresholding of fMRI data

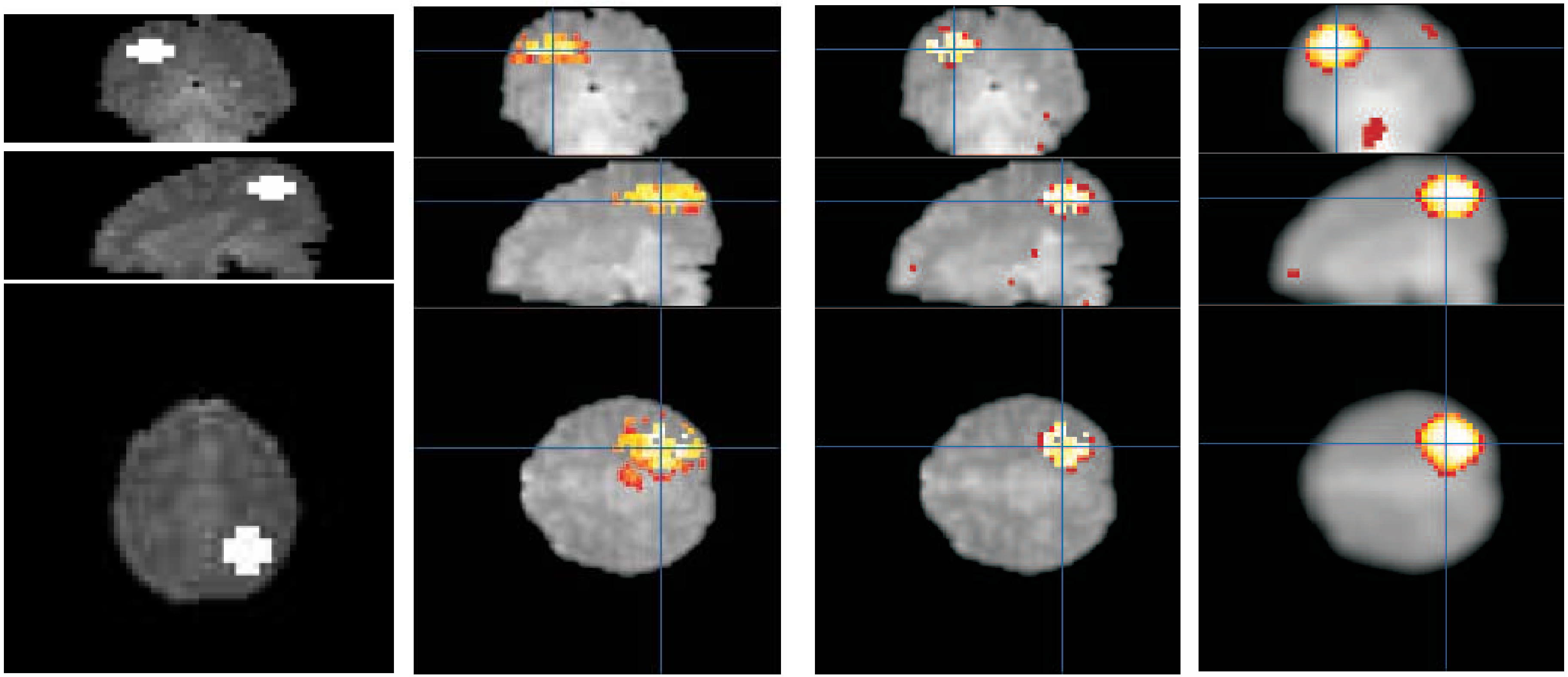


Noise suppression, using probability of detail presence

- Wavelet decomposition
- Creation of the detail mask (by comparing original image and a supposed noise-free image)
- Computing the probability of detail presence along the mask
- Shrinking the coefficients regarding the probabilities
- Wavelet reconstruction

Examples

Source Wavelet shrinkage Gaussian smoothing

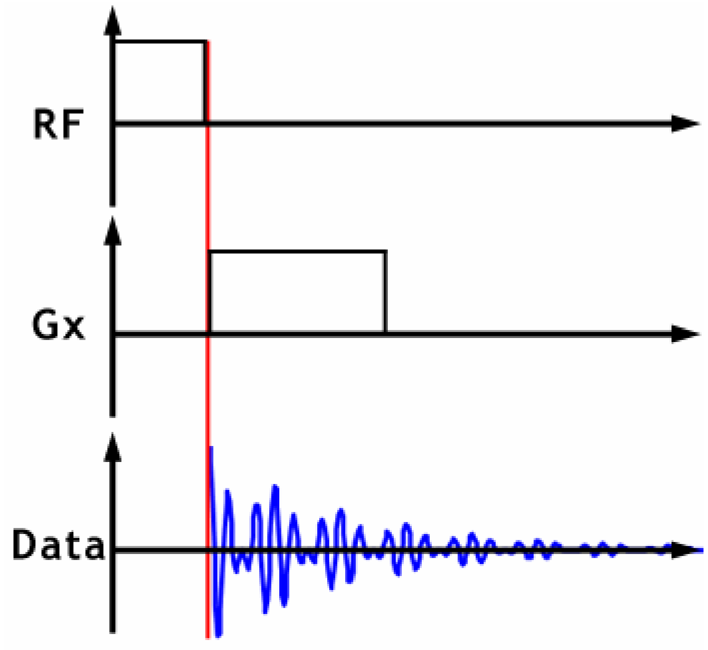


Wavelet MRI encoding

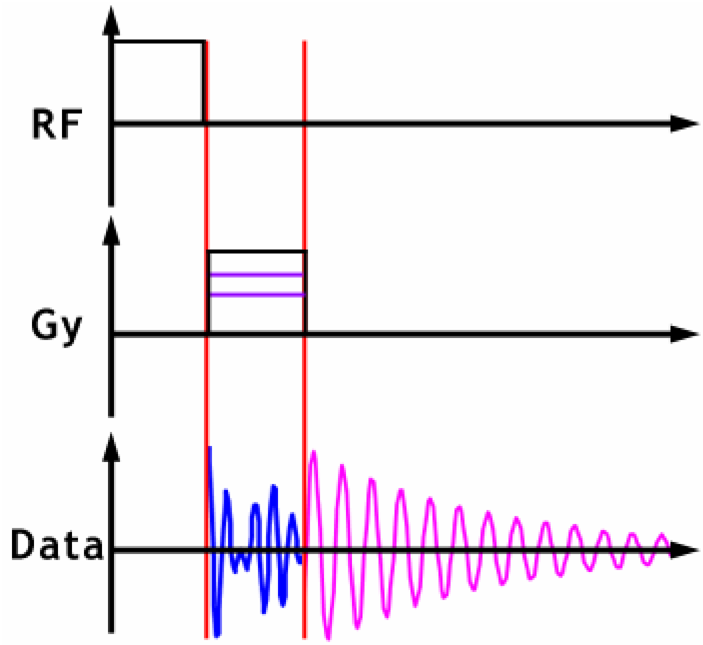
- Common MRI methods
- Wavelet encoding

Encoding of spin distribution.

- Frequency



- Phase

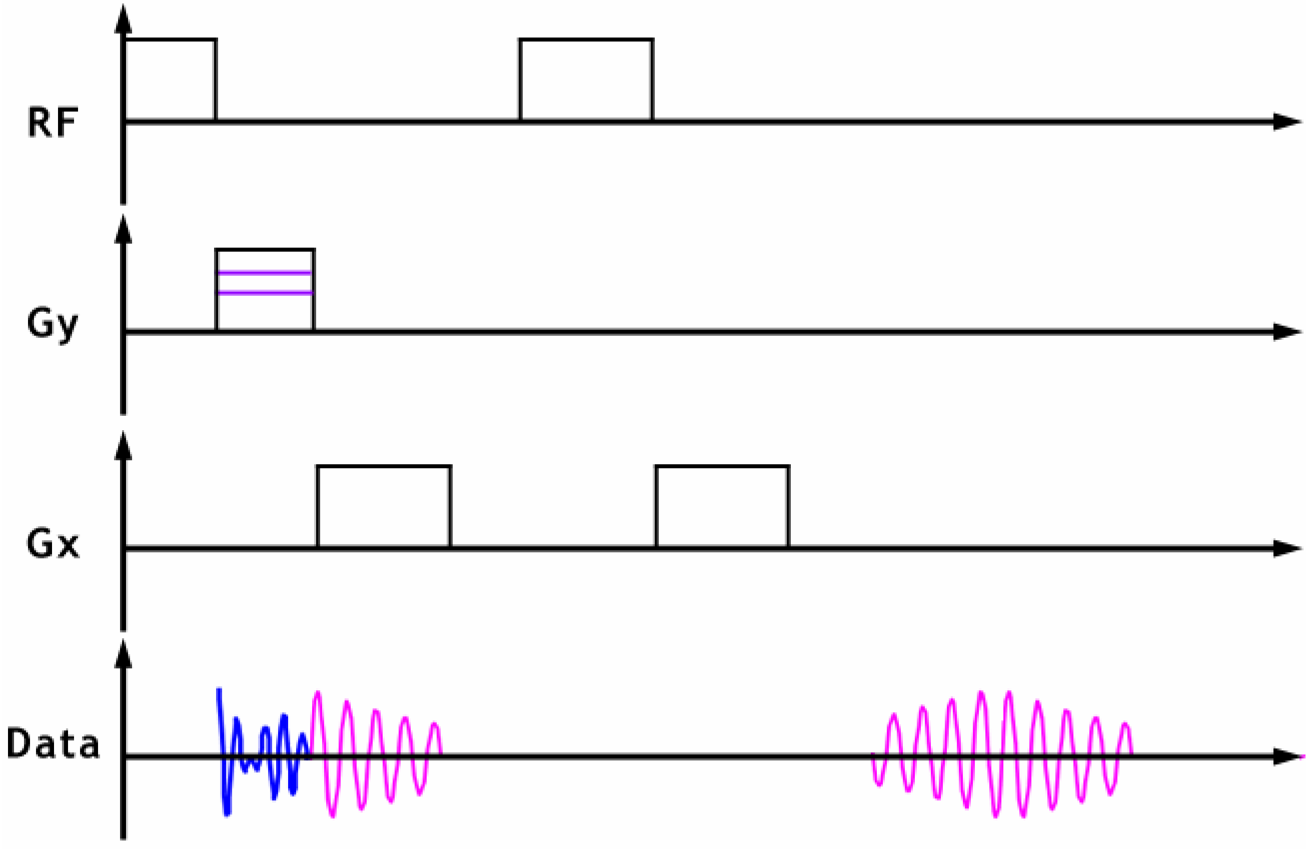


Idea of Fourier method

- Fourier encoding consists of acquiring FID on one direction (frequency encoding) and a FID analogue (phase encoding) on another.
- After that Fourier transform in each direction is applied to get image from the source data.

Fourier spatial encoding

- Phase (y) + frequency (x)

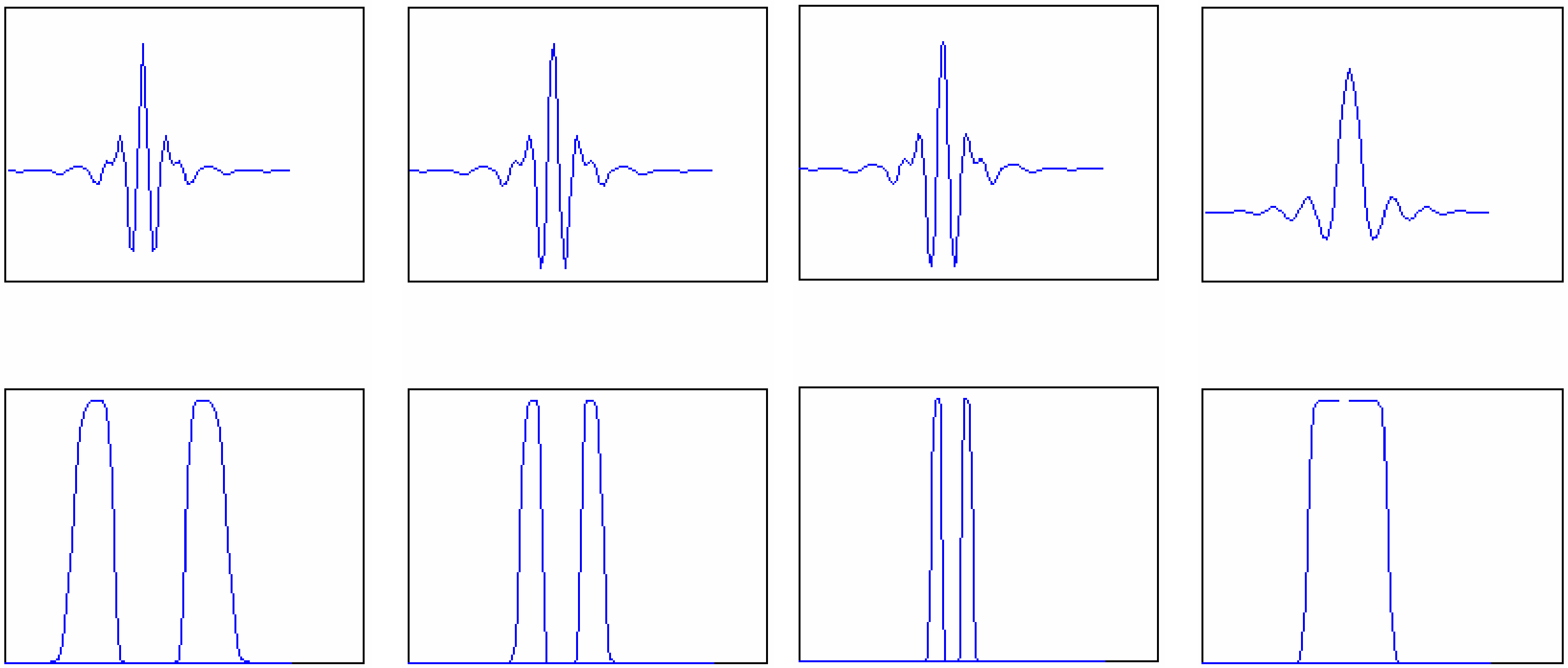


Idea of wavelet method

- As in Fourier encoding it is wise to acquire a 1D distribution through FID and Fourier transform (frequency encoding).
- The other dimension is scalar multiplied (in frequency domain) by wavelet basic functions, thus acquiring wavelet coefficients.
- Fourier and wavelet transforms are applied to the data in different directions to get an image.

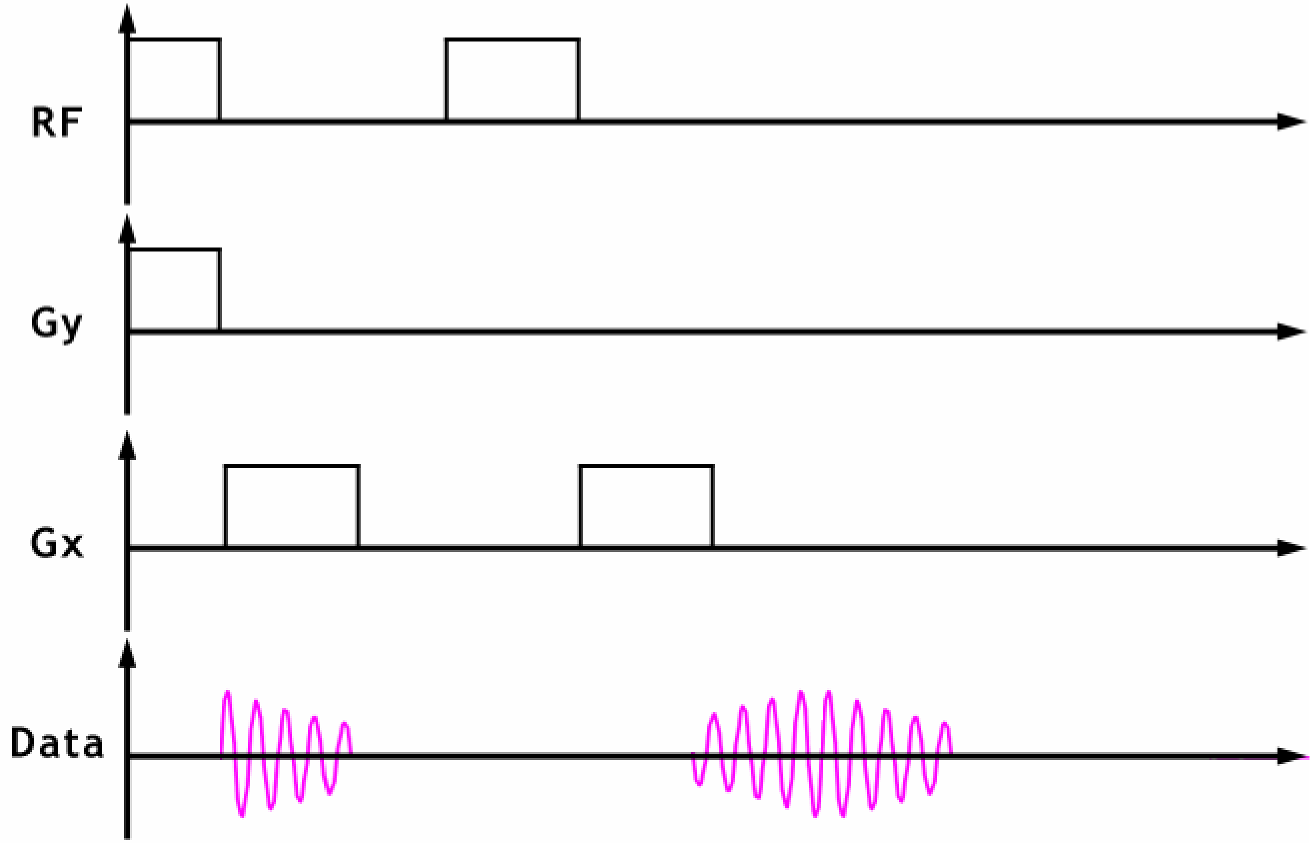
Wavelet encoding basics

- Basic functions spectra – pulse envelope

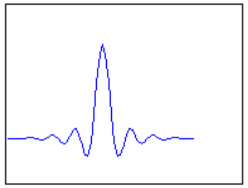


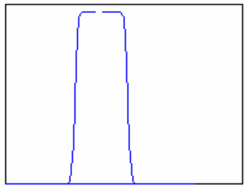
Wavelet spatial encoding

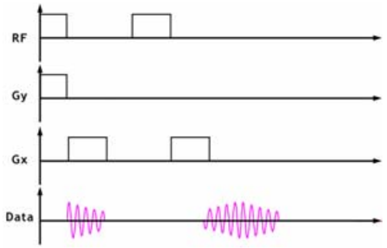
- Wavelet (y) + frequency (x)

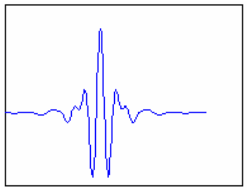


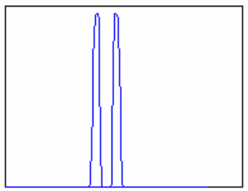
Experiment sequence

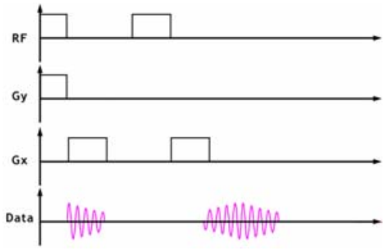












Fourier transform

+

Wavelet transform

Image

Examples

Fourier encoding

wavelet encoding

wavelet encoding (only 1/3 of original data)