## INTRODUCTION

This paper is about the algebraic surfaces of a higher degree. These surfaces are a very efficient method to describe various objects of nature as well as artificial objects. These surfaces posses such are smooth, continuous and differentiable. Most important, algebraic surfaces can be recorded in a compact manner: one needs to write only a list if coefficients of a general algebraic equation of a given order. For an algebraic surface of three variables of the third order we need to record only 20 values, for surface of fourth order only 35 values need to be recorded. If we use a floating point representation of a real number, we need only 4 bytes per coefficient. This means that data on surface of the third order can be recorded and transmitted only in 80 bytes, and quartic surface – in 140 bytes. As it will be shown later in this paper, surfaces if third and fourth order already provide a large number of complicated unique geometric forms, and one uses a combination of several surfaces – even more complex scenes can be recorded and transmitted. This feature makes this form of representation very useful in telecommunications where bandwidth is almost always not big enough and is very costly. Algebraic equations of three variables are most commonly used, but other variables can be added into equation representing colour, temperature, density, tenacity, rigidity, strength and other physical parameters, where each point of the surface has a corresponding value(s) of this parameter.

The methods of representation of 3D graphical information is considered a very hot topic in today's research and development. As mobile devices become more powerful and capable of complex data processing the bandwidth often becomes a bottleneck for transmission of graphical data. Raster and vector graphics take a lot of bandwidth and often times may not be used in a real-time applications in the field devices. Therefore there is a growing interest in other methods of mathematical representation of 3D objects. In one-two years hi-end mobile phones (smart-phones) are likely to have embedded VRML browsers. Several mobile phones manufacturers are conducting their development of such devices. VRML standard can contain a definition of algebraic surfaces, i.e. representation of surfaces by algebraic surfaces is supported by this standard. The real question is will encoders use algebraic surfaces of higher degree for generating scenes.

There are many areas where algebraic surfaces can be used. In games on mobile devices algebraic surfaces can be used for representation of 3D objects. If mobile devices could quickly exchange this information there could be games with several players playing in the same game, just like in regular PC games played by several people on several personal computers connected into network. Another area of deployment is telemedicine. In telemedicine the time factor is critical therefore data needs to be represented in the most compact way yet containing all important details. Algebraic surfaces can achieve both of these goals.

The wide use of algebraic surfaces has been restricted in the past by the lack of complete classification of these surfaces. The only classification that we know is the classification a quadric surfaces. This classification uses four matrixes composed of values of coefficients of the equation. The correlation of the determinants of these matrixes allows to ascribe given surface to one of 17 known types. The existence of such complete classification for quadrics allows their use in CAD systems for approximation of real surfaces as NURBS – Not-Uniform Rational B-Splines. If there was a classification of surfaces of third and forth degrees – they could be used just like NURBS are being used today.

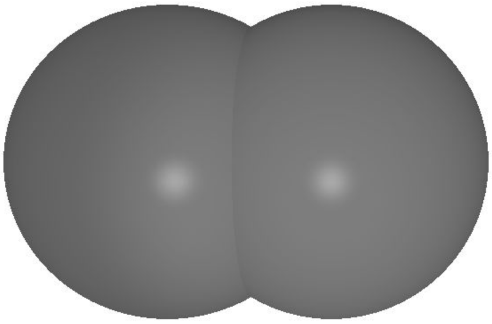
In this paper there is a description of a new approach to widen the number of classes of surfaces to be used in applications.

## METHODS OF OBTAINING NEW SURFACES OF 3RD AND 4TH DEGREES

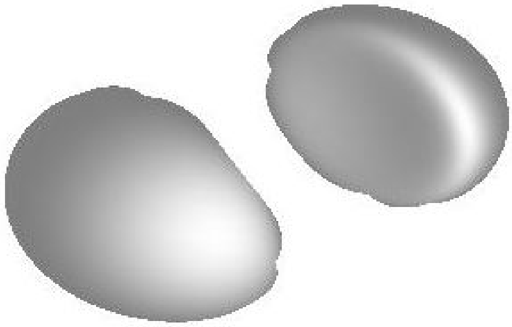
### Surfaces obtained by combining two surfaces of lower order

New surfaces of the fourth order can be obtained by combining two quadrics, or by combining a know surface of the third order with a plane. Among "known" surfaces of the third order there are those that we just derived from the surfaces of the second order (quadrics). For instance, here is a surfaces composed of two intersecting spheres of radius two with centres in x=-1 and x=1:

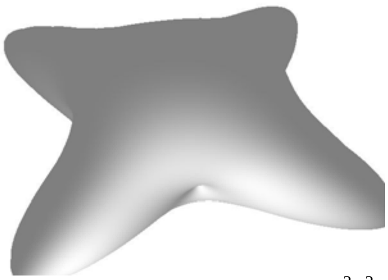
$x^4 + 2x^2y^2 + 2x^2z^2 - 10x^2 + y^4 + 2y^2z^2 - 6y^2 + z^4 - 6z^2 + 9 = 0$



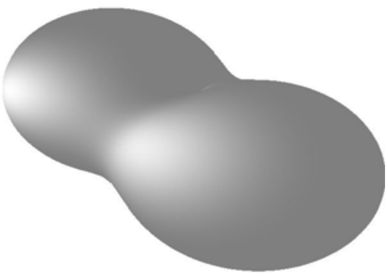
Surface $x^4 + 2x^2y^2 + 2x^2z^2 - 10x^2 + y^4 + 2y^2z^2 - 6y^2 + z^4 - 6z^2 + 9 = 0$ ("Two spheres")



"two spheres" with a free coefficient set to -15



"two spheres" with a coefficient next to $x^2y^2$ set to -1



"two spheres" with a coefficient next to $y^2$ set to -34

These are only a few types, further adjustment of coefficients can fine-tune the ratio of concavity, convexity, oblongness, etc. These modifications can’t be obtained by simple operations: enlarge/shrink, because those modify all coefficients in one direction and here tuning is done one by one coefficient individually.

### Surfaces obtained by setting coefficients of equation to -1, 0 and 1

Changing coefficients of a general equation of the surface of the forth degree written in a standard lexicographical form provides a vast variety of new forms of surfaces. Even if we consider only those equations, that have a34 through a0 set to -1, 0 or 1 we get 50 031 545 098 999 707 individual surfaces. The real question here is how to make sure that a new surface is truly new and was not included into a library before. Today even for 2D pictures there is a problem how to search and compare two pictures or how to find closest match of a given picture in a database of pictures. This problem is not solved yet. The problem with comparing 3D shapes is even more complex. To ensue that a given surface is a new one we need to prove that it can not be obtained from the other ones by operations of expansion, move, turn, etc. For this very reason we took only value 1, 0 and -1. It can be proved that changing coefficients at radicals 34 through 20 for equation of forth order and leaving the rest of coefficients unchanged provides new surfaces. So, it ban be shown that we can obtain at least 14348907 new surfaces such that one can not be transform to another by operations of linear move or turn around any axis.
To demonstrate this principle consider a shape of the second order:

Fig 8. One sheeted hyperboloid $x^2 - y^2 - z^2 +1 = 0$

That we set a free coefficient to 0 and get a new shape::

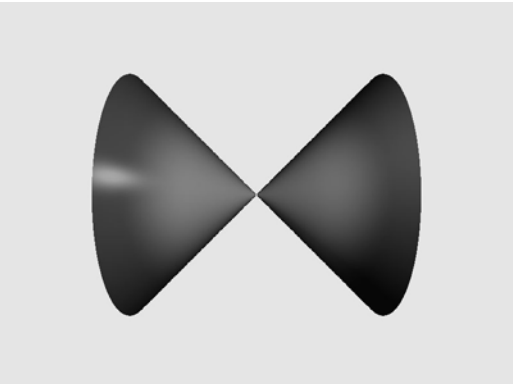


Fig 9. Elliptic cone $x^2 - y^2 - z^2 = 0$

That we set a free coefficient to to -1 and get a new shape once again:

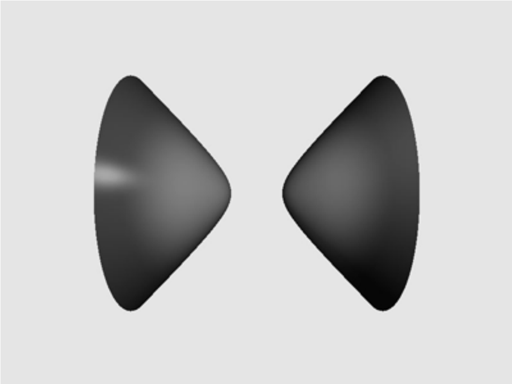


Fig 10. Two-sheeted hyperboloid $x^2 - y^2 - z^2 - 1 = 0$

In this example the difference is obvious. It so not always that easy. So there is a search of the algorithm that could help to identify a given surface in a library, we need to find or develop and algorithm that could compare two surfaces with operations of rescaling in all directions and all possible rotations.

## CONCLUSIONS

These three new methods for obtaining new surfaces allow to obtain a large variety of new cubic and quartic surfaces. Perhaps not all possible quartic are obtained via these methods but the number of such surfaces is huge. A complete classification of all surfaces of the fourth degree is not likely to be finished in the near future. However using some classes or quartic and cubic surfaces can already enhance modern systems for modelling real-life objects. Later other classes of these surfaces can be studied and also added to these systems. Step by step we can widen the range of surfaces to be used in application making use of this powerful tool. There are other tasks related to this subject. One of them is how to sub-classify the surfaces obtained by these three methods. Method 2.4 provides almost one and a half million new surfaces. Another task is how to search Changing coefficients of a general equation of the surface of the forth degree written in a standard lexicographical form provides a vast variety of new forms of surfaces. Even if we consider only those equations, that have a34 through a0 set to -1, 0 or 1 we get 50 031 545 098 999 707 individual surfaces. Changing coefficients within this group of surfaces can modify, turn, expand or shrink a given surface. However it can be shown that we can obtain at least 14348907 new surfaces such that one can not be transform to another by operations of linear move or turn around any axis.