JOINT ADVANCED STUDENT SCHOOL 2008, ST. PETERSBURG

FORMATION CONTROL FOR MULTI-ROBOT SYSTEMS

Final Report by

cand. ing. Aubery Marchel Tientcheu

born on 04.01.1986
address:
Rambergstr. 6
80799 München
Tel.: 089 38394957

Institute of

AUTOMATIC CONTROL ENGINEERING

Technische Universität München

Univ.-Prof. Dr.-Ing../Univ. Tokio Martin Buss
Univ.-Prof. Dr.-Ing. Sandra Hirche
Univ.-Prof. Dr.-Ing.. Olaf Stursberg

Contents

1   Introduction

With the rapid development of artificial intelligence, autonomous robots, sensors, automatic control, and computer vision systems, multi-robot systems have been widely applied to various real-world applications such as search and rescue, surveillance, collaborative exploration, and exploration of large and unmapped areas. One of the most important aspects, which have been search, is the formation control in multi-robots systems. The research in this area was inspired by biological phenomena like flocks of birds in flight. A very famous model, the so-called boids was developed by Reynolds, where each bird has his how strategy of movement and only local information available. However a collective behaviour is obtained. This strategy has three components: separation, steer to avoid crowding; alignment, steer toward the average heading of neighbors; cohesion, steer toward the average position of neighbors. Using these biological phenomena, we can use them to control real technical systems, in order to achieve formation in multi-robot systems. In this report, we will study the agreement or rendezvous problem. The main goal is to find a control law such that all robots converge to a common position by using only local sensor information. In the first part, we will introduce the formation in cyclic pursuit for the fully actuated point of mass model und especially stabilization to a point, then the stabilization to more general geometric pattern. Then we will show conditions under which we can avoid collisions in this system. In the second more realistic case, we will study the formation for unicycle and local information controller with which we can achieve convergence.

2   Formations in cyclic pursuit.

2.1 Model Description

We considered n numbered robots modelled as points in complex plane: $z_1,\ldots,z_n$ with the kinematic model:

$$\dot{z}_i = u_i.$$

(2.1)

We assume that the robots are fully actuated that mean they can move in all directions. But this is a restriction to the reality since "real" robots can not move in all directions.

Let us now assume that robot I is able to measure the position of robot i+1. We then get the following equations for the output of the system:

$$y_i = z_{i+1} - z_i, i = 1,...,n - 1$$

$$y_n = z_1 - z_n.$$

(2.2)

For this system, we define the centroid of the points $z_1,...,z_n$ as the point z_0 with:

$$z_0 = (\frac{1}{n})\sum_{i=1}^{n} z_i$$

(2.3)

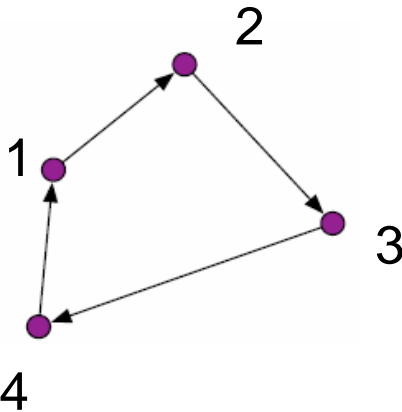The control law is steer over toward the next robot (closed loop controller). Figure (2.1) shows the case n=4.

Figure 2.1: Cyclic Pursuit

That mean:

$$\dot{z}_i = u_i, \ i = 1, ..., n$$
$$u_i = F_i y_i, \ F_i = 1$$
$$\textit{Then follows}: \ \dot{z}_i = z_{i+1} - z_i$$
$$E.g.: \textit{for} \ n = 4$$

$$\begin{cases} \dot{z}_1 = z_2 - z_1 \\ \dot{z}_2 = z_3 - z_2 \\ \dot{z}_3 = z_4 - z_3 \\ \dot{z}_4 = z_1 - z_4 \end{cases} \quad \dot{z} = Mz, \textit{with} \quad M = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \qquad (2.4)$$

2.2 Stabilization to a point

For the previous model, we assume that the matrix M has almost one eigenvalue at the origin since the rows sum of M is zero.

The other eigenvalues of M have negativ real parts.

Then it follows that for initial locations of the agents, the centroid of the point is stationary and every z_i converge to this centroid (agreement problem).

That mean we achieve convergence of all the robots to a point.

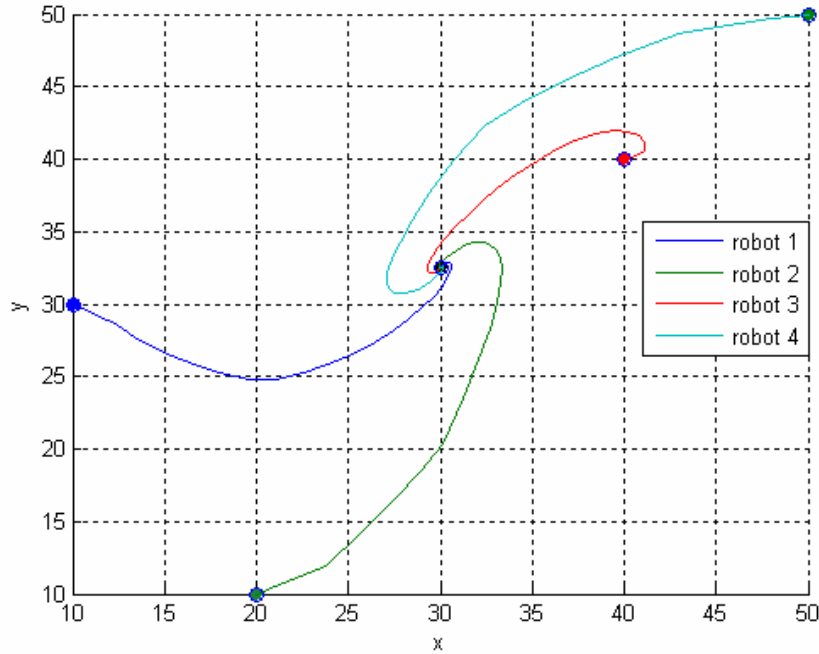The following simulation shows the behaviour of four robots:

Figure 2.2: Convergence in a four robot system

2.3 Stabilization to general geometric pattern

In the last paragraph, we considered that each robot pursues the next robot position and we achieve convergence to a point. In this case the points converge to a common location, namely to a point.

In this following paragraph, we considerer that each robot pursuit the position of the next position, displaced by a certain vector.

$$\dot{z}_i = (z_{i+1} + c_i) - z_i, \qquad i = 1,...,n-1,$$
$$\dot{z}_n = z_1 + c_n - z_n$$
$$\dot{z} = Mz + c$$

(2.5)

Let us assume that, d is the unique vector satisfying: Md + c = 0. Then it follows that:

$$\dot{z} = M(z - d)$$

$$(2.6)$$

We have reduced our problem to the convergence to a point and we can formulate the following theorem:

Concerning the previous system, assume the centroid of the points $c_1, \ldots, c_n$ is at the origin. Let d denote the unique vector satisfying $Md + c = 0$. Then for every n initial positions of the robots, the centroid of the points $z_1, \ldots, z_n$ is stationary and robot i converges to this centroid displaced by d_i.

The following simulation shows an example of convergence to a triangle for 6 robots:
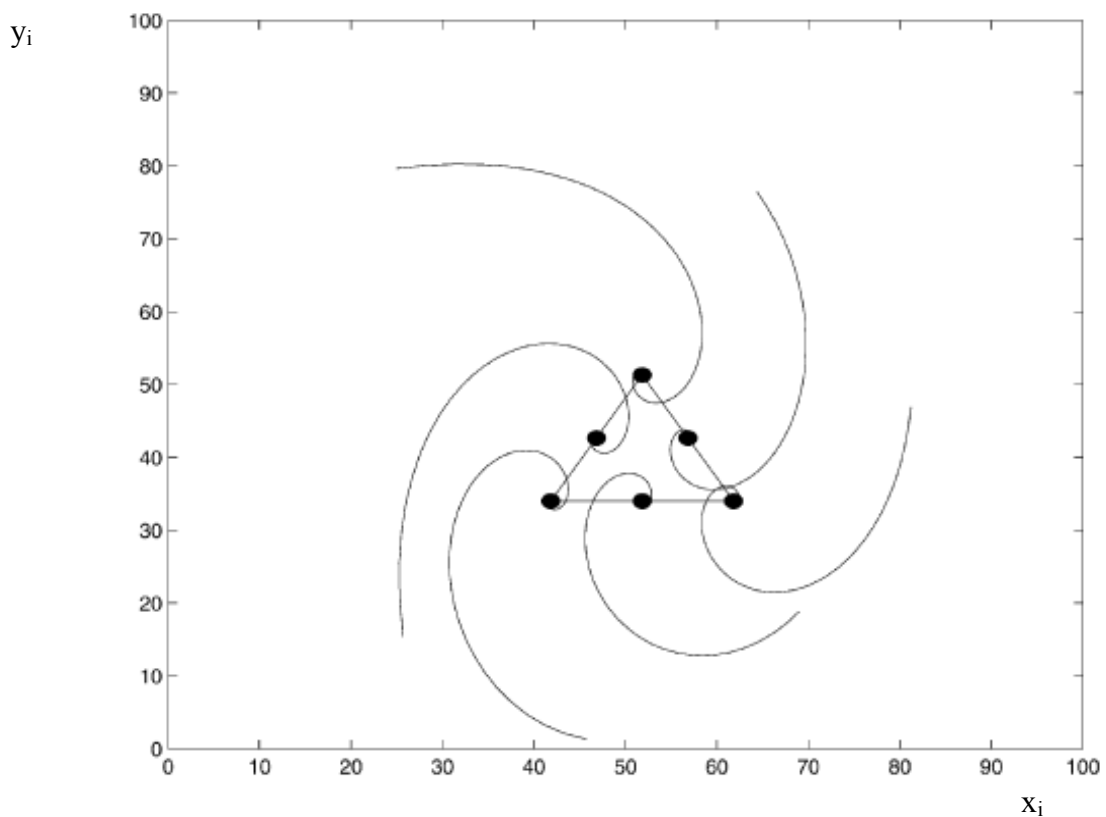

Figure 2.3: Convergence of a 6 robot-system to a triangle

2.4 Collision avoiding

In the last part, we study convergence to a general geometric pattern, but was is with collision? We didn't make sure that the robots do not collide during the formation. Before we take this in consideration, let us define a counter clockwise star formation.

2.4.1 Definition

N point are said to be arranged in a counter clockwise star formation

if $r_i > 0$ and $\alpha_i > 0$ for all i = 1, . . . ,n and $\sum_{i=1}^{n} \alpha_i = 2\pi$.
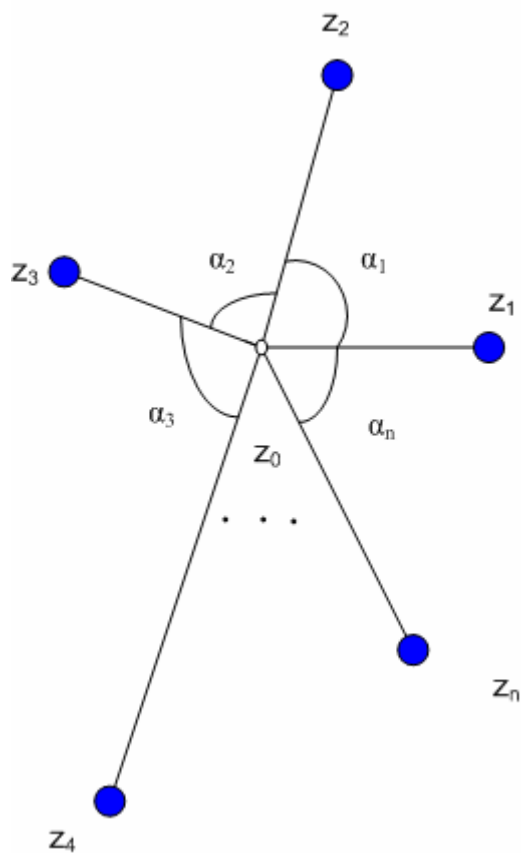

Figure 2.4: Points in counter clockwise star formation

Now let us come back to collision avoidance. In [1], the following theorem is formulated as a necessary condition for collision avoidance:

2.4.2 Theorem

Suppose n > 2 distinct points initially are arranged in a counter clockwise

star formation. Under cyclic pursuit they remain in a counter clockwise star formation. (In particular, they never collide).

We have now proved that we can achieve converge to general geometric pattern with a cinematic point model of the robots; we can also avoid collisions. But in this case we assume that we have fully actuated robots; that mean the robots can move in all directions. This case is not really realistic, since real robots can not move in all direction. For this reason, we introduce the unicycle model.

3 Formations in unicycle

3.1 Model description

We now consider a group of n wheeled vehicles also called unicycle. Each unicycle(See Figure 3) in the (x,y) plane with the heading angle θ. It can only move in the direction of its heading. This constraint is called a nonholonomic constraint und can be expressed by:
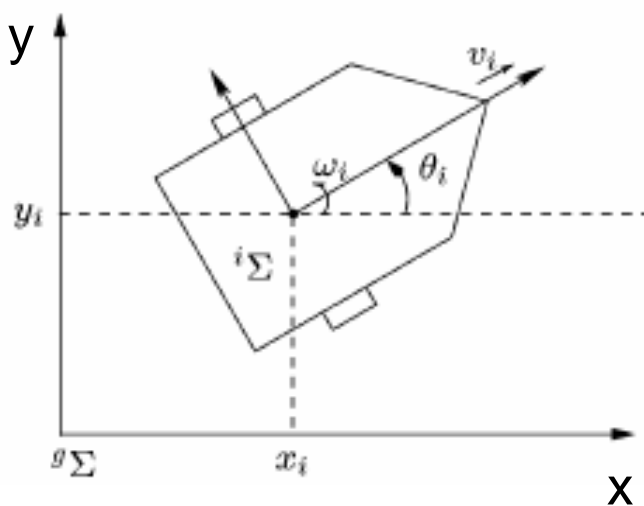
$(x, y) \perp (\sin(\theta), -\cos(\theta))$.

Figure 3.1: Unicycle

Each unicycle is can be describe by:

$$\begin{cases} \dot{x}_i = v_i\cos(\theta_i) \\ \dot{y}_i = v_i\sin(\theta_i) \\ \dot{\theta}_i = \omega_i \end{cases} \quad or \quad \begin{cases} \dot{z}_i = v_i e^{j\theta} \\ \dot{\theta}_i = \omega_i \end{cases}$$

Where v_i is the velocity of the unicycle.
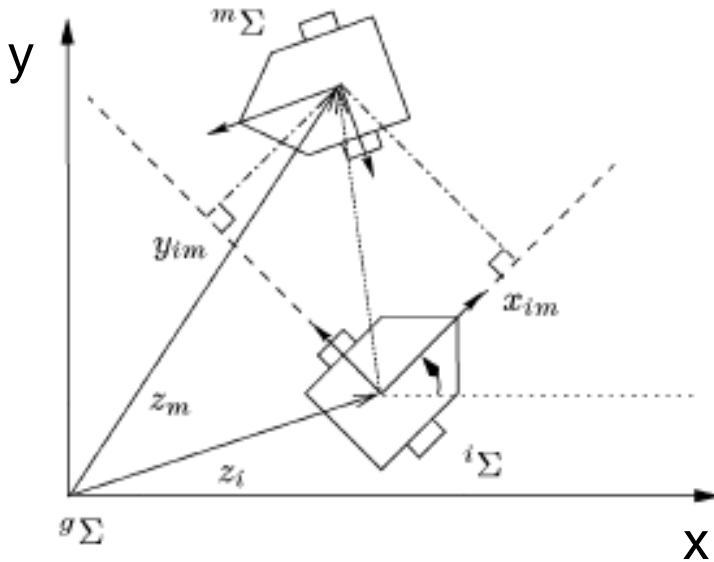
We then construct a

Figure 3.2: Frenet-Serret Frame

We construct a moving frame $^i\Sigma$, the Frenet-Serret frame, that is fixed on the vehicle. Let r_i be the unit vector tangent to the trajectory at the current location of the vehicle and let s_i be r_i rotated by $\pi/2$.

Thus, $\dot{z}_i = v_i r_i$ since the vehicle is moving at the speed v_i.

Since vehicle i can only get the relative positions of a subgroup Ni of vehicles with respect to its own Frenet-Serret frame:

$$\begin{cases} x_{im} = (z_m - z_i) \cdot r_i \\ y_{im} = (z_m - z_i) \cdot s_i \end{cases} \quad m \in N_i$$

Before we speak about the feasibility of convergence in the unicycle model, let us start with some definitions and background.

3.2 Local information controller

A controller (v_i, w_i), i=1,…,n, is said to be a local information controller if:

$$\begin{cases} v_i = g_i(t, x_{im}, y_{im}) \,|\, m \in N_i \\ w_i = h_i(t, x_{im}, y_{im}) \,|\, m \in N_i \end{cases} \quad i = 1,...,n$$

where g_i is such that $\left\{ (\forall m \in N_i) \; z_m = z_i \right\} \Rightarrow \left\{ v_i = 0 \right\}$

3.3 Background about graph theory

A graph G = (V,E) is defined to be a pair of sets V and E. V is the set of a limited number of nodes and E is the set of arcs.

There are two important types of graphs: the directed graph (digraph) where the links between the nodes are just in one direction and the undirected graph where the links are in the both directions.

Several matrix can be associated with a directed graph:

- The adjacency matrix A = (aij): If there is an arc from node i to node j then the entry aij equals 1, otherwise 0.

- The degree matrix D = (dij) is defined to be a n×n diagonal matrix where the diagonal entry dii is the out-degree of node i.

- Finally the graph Laplacian is the matrix defined as L=D−A.

If there is a path in G from one node i to another node j, then j is said to be reachable from i written. If not, j is said to be not reachable from i.

If a node is reachable from every other node in G, then we say it is globally reachable.

A sensor digraph (directed graph) models the information flow, where a link from node i to node j indicates that vehicle i can sense the position of vehicle j.

3.4 Problem Statement

We now come back to our main topic, namely is it possible to achieve convergence for unicycle? The problem can be formulated in this way:

Find a local information controller such that for all $(x_i(t_0), y_i(t_0), \theta_i(t_0)) \in R^3$

i=1,…,n and for all t_0:

$$(\exists z_{ss} \in R^2) \, (\forall i) \, \lim_{t \to \infty} z_i(t) = z_{ss}.$$

3.5 Solution

The information flow is central to solve the previous problem since the vehicles are coupled through the sensor information flow.

In fact the information flow produce a kind of symmetry in the systems equations with respect to the x and y coordinates.

It can be proven that (see [2]), the problem of convergence can be solved if and only if the graph has a globally reachable node. An algebraic characterization of this property is that, the Laplacian L of the graph has a simple eigenvalue 0.

Moreover, we can find a local information controller, so that convergence is achieved. The proof is constructive.

The idea is that each robot use a control strategy based on his knowledge of the position of his neighbors. As we have seen it before, an unicycle i can only get the relative positions of a subgroup Ni of vehicles with respect to its own Frenet-Serret frame

$$\begin{cases} x_{im} = (z_m - z_i) \cdot r_i \\ y_{im} = (z_m - z_i) \cdot s_i \end{cases} \quad m \in N_i$$

We then use the controller:

$$\begin{cases} v_i(t) = k \sum_{m \in N_i} x_{im} \quad k > 0 \\ \omega_i(t) = \cos(t) \end{cases}$$

The robot i follows the position of his neighbors and his velocity is $v_i(t)$:. He also turn permanently around his own center, so that his velocity is not zero.

As we can see in the example below, $x_{im}=0$, when the axes r_i of the two robots are parallel. That mean $v_i = 0$. If the robots did not permanently turn around their axes using $\omega_i(t)$, they would not move.
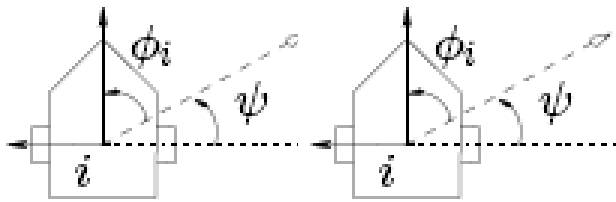
Figure 3.3: robot axes are parallel. No information available

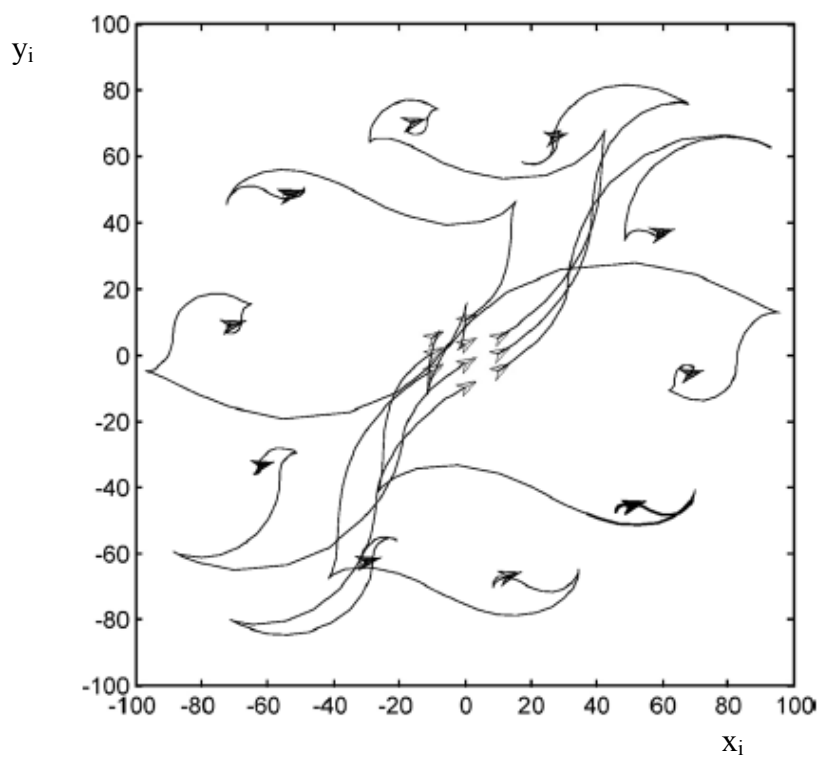The following simulations show the convergence to a cycle:

Figure 3.4: Convergence to a cycle

4 Summary

In this report, we investigated the formation control properties in multi-robots systems. We first started with the cyclic pursuit case and then study the feasibility of convergence for the more realistic unicycle model. We have given necessary and sufficient conditions for convergence and to avoid collisions in these systems. Furthermore we developed a constructive method to find out a controller for the system. Further researches treat the question of, how to provide better controller, who do not keep the unicycles wiggling.

Bibliography

[1] Zhiyun Lin, Mireille Broucke, and Bruce Francis, "Local Control Strategies for Groups of Mobile Autonomous Agents" IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 49, NO. 4, APRIL 2004

.[2] Zhiyun Lin, Bruce Francis, and Manfredi Maggiore, "Necessary and Sufficient Graphical Conditions for Formation Control of Unicycles" IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 50, NO. 1, JANUARY 2005

.[3] Johannes Dold, "DIGITAL CONTROL OF MOBILE ROBOTS IN FORMATION", Diplom Thesis,January 3, 2008

[4] Bruce Francis, „Distributed Control of Autonomous Mobile Robots", ECE1635 Course Notes, Version 1.4, September 2007

[5] http//www.control.utoronto.ca