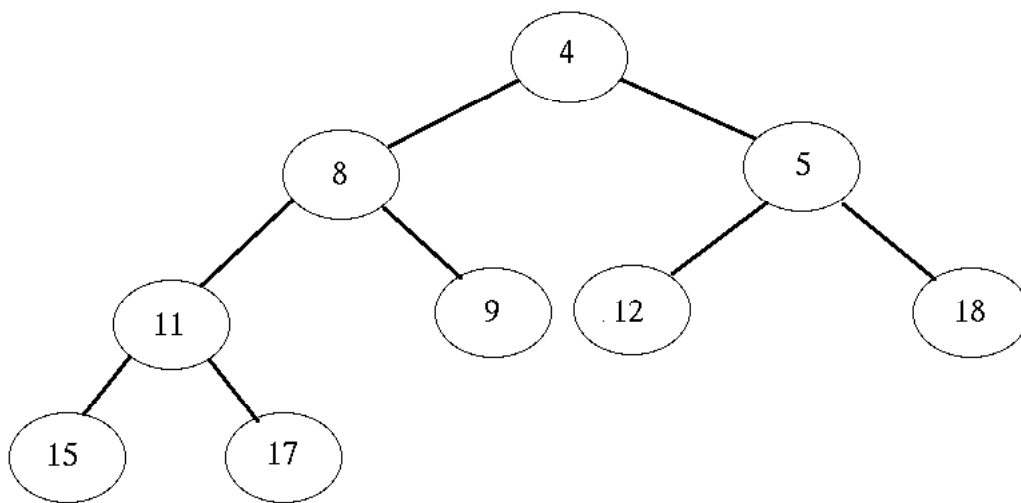

Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1

Consider the following Binary Heap:



Carry out the operations in the following order and show, after each operation, what the Binary Heap looks like (always carry out each operation on the result of the previous operation):

1. deleteMin()
2. insert(3)
3. deleteMin()
4. insert(10)

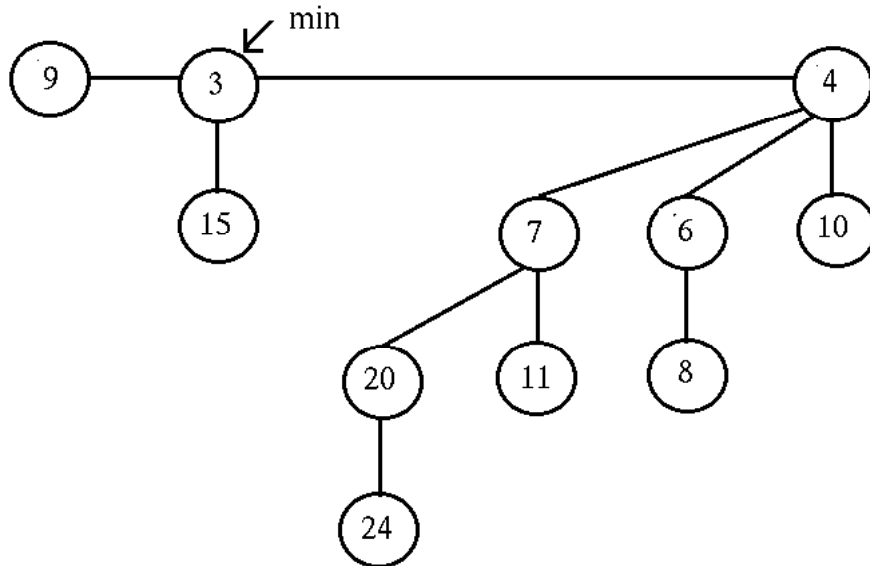
Aufgabe 2

We discussed that the delete operation for Binary Heaps takes $O(\log n)$ time, however we did not discuss how this operation works. Describe how this algorithm would be implemented (or give a pseudo-code algorithm) such that it is correct and that the runtime is $O(\log n)$.

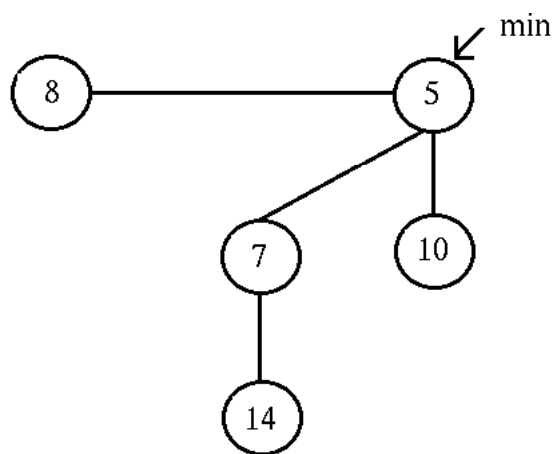
Aufgabe 3

Consider the following Binomial Heaps:

Heap A:



Heap B:



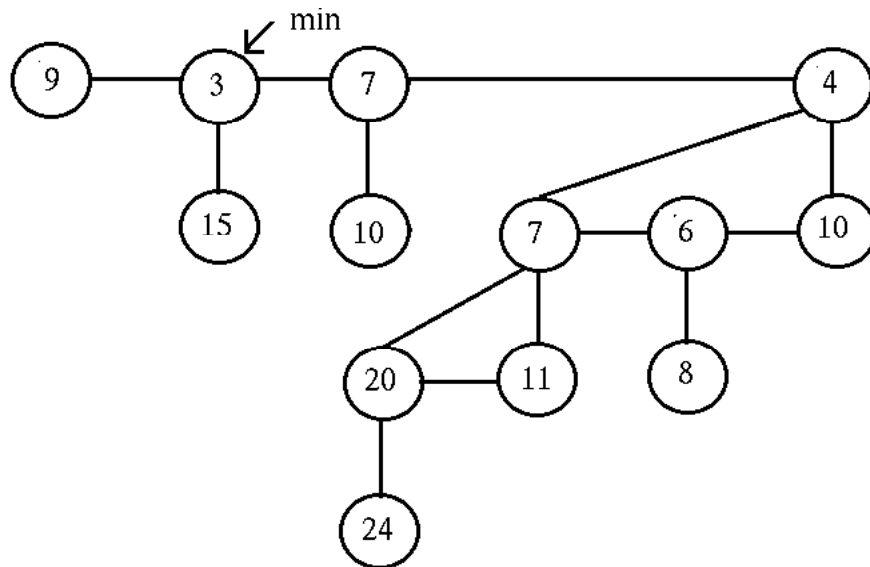
Carry out the operations in the following order and show, after each operation, what the Binomial Heap looks like (always carry out each operation on the result of the previous operation):

1. merge(A,B)
2. deleteMin()

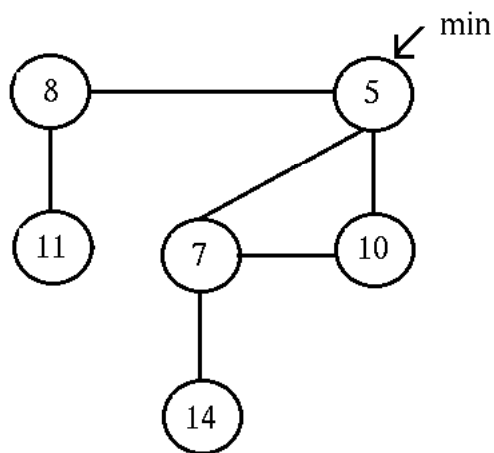
Aufgabe 4

Consider the following Fibonacci Heaps:

Heap A:



Heap B:



Carry out the operations in the following order and show, after each operation, what the Fibonacci Heap looks like (always carry out each operation on the result of the previous operation):

1. merge(A,B)
2. insert(22)
3. deleteMin()
4. delete(6)

Aufgabe 5

In discussing the Radix Heap, we also discussed the $msd(k_{min}, k)$ function. Assuming $k_{min} = 9$, solve $msd(k_{min}, k)$ for the following values of k :

1. $k = 9$
2. $k = 13$
3. $k = 24$
4. $k = 50$

Aufgabe 6

Consider the $deleteMin()$ operation for Radix Heaps. Assume that k_{min} is the element to be removed, k'_{min} is the smallest element after k_{min} has been removed, and i is the smallest i s.t. $B[i] \neq \emptyset$ after k_{min} has been removed. We said that the following property holds:

$$\forall_{j>i} \forall_{k \in B[j]} msd(k_{min}, k) = msd(k'_{min}, k) = j$$

Please explain why this is the case.