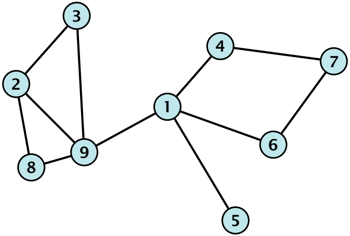
16 Global Mincut

Given an undirected, capacitated graph $G = (V, E, c)$ find a partition of V into two non-empty sets $S, V \setminus S$ s.t. the capacity of edges between both sets is minimized.



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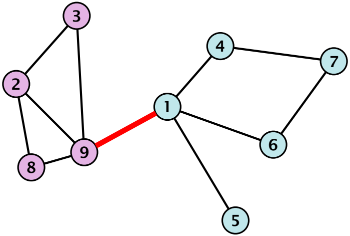

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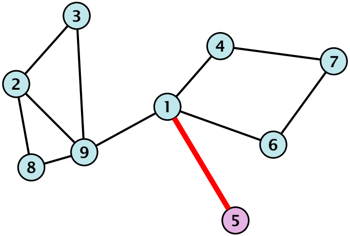

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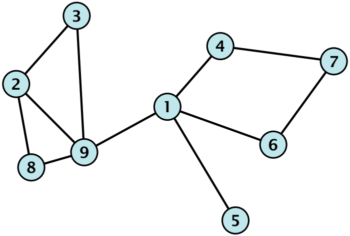

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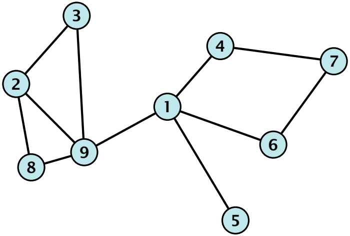

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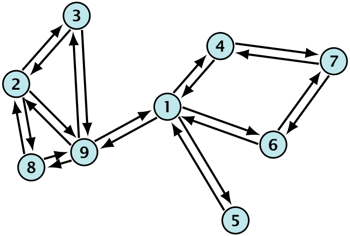
- Construct a directed graph $G' = (V, E')$ that has edges (u, v) and (v, u) for every edge $\{u, v\} \in E$.



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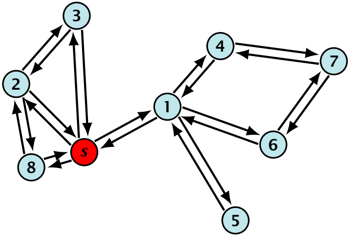

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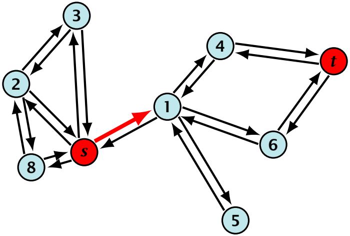

Edge Contractions

- Given a graph $G = (V, E)$ and an edge $e = \{u, v\}$.
- The graph G/e is obtained by "identifying" u and v to form a new node.
- Resulting parallel edges are replaced by a single edge, whose capacity equals the sum of capacities of the parallel edges.

Example 88

- Edge-contractions do no decrease the size of the mincut.

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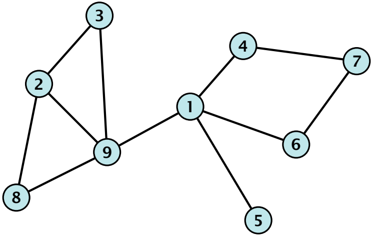

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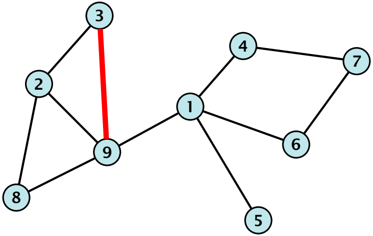

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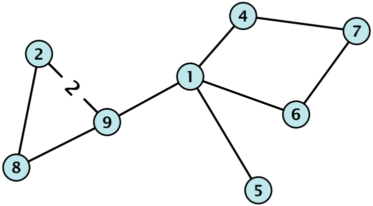

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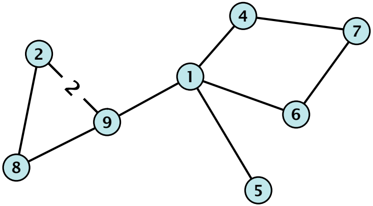

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Edge Contractions

We can perform an edge-contraction in time $\mathcal{O}(n)$.

Randomized Mincut Algorithm

Algorithm 52 KargerMincut($G = (V, E, c)$)

```
for $i = 1 \to n - 2$ do
    choose $e \in E$ randomly with probability $c(e)/C(E)$
    $G \leftarrow G/e$
return only cut in $G$
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- Note that the final graph G_2 only contains a single edge.
- The cut in G_2 corresponds to a cut in the original graph G with the same capacity.
- What is the probability that this algorithm returns a mincut?

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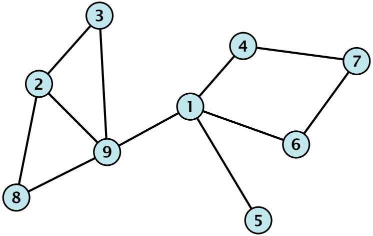

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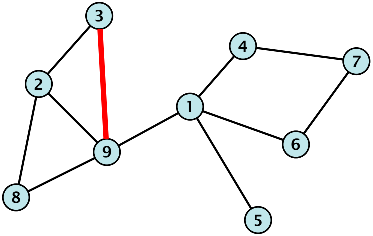

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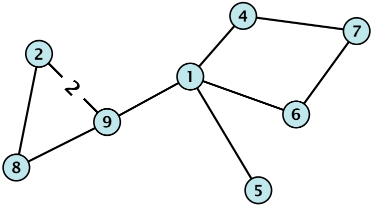

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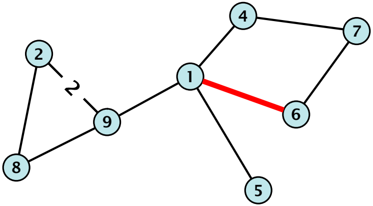

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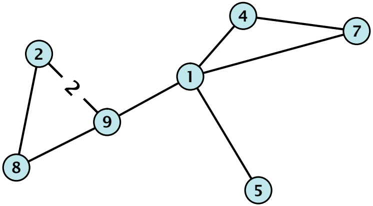

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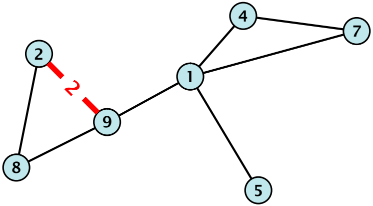

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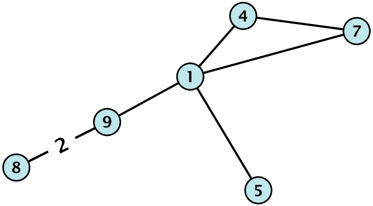

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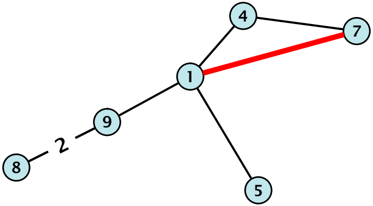

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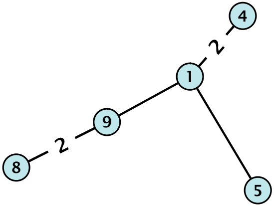

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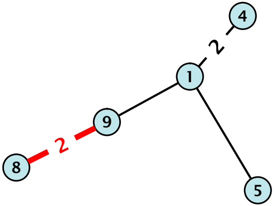

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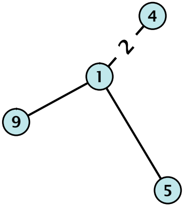

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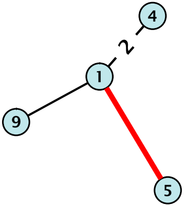

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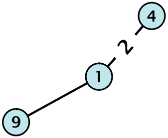

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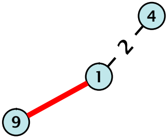

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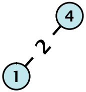

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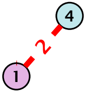

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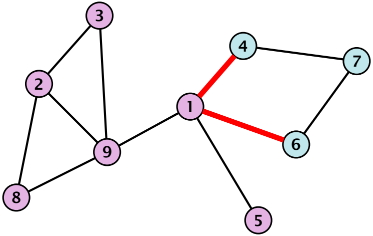

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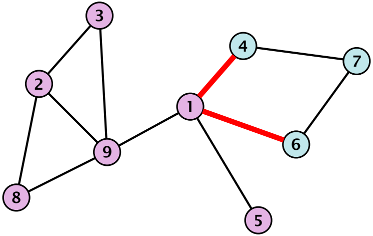

What is the probability that this algorithm returns a mincut?

Analysis

What is the probability that a given mincut A is still possible after round i?

- It is still possible to obtain cut A in the end if so far no edge in $(A, V \setminus A)$ has been contracted.

Analysis

What is the probability that we select an edge from A in iteration i?

- Let $\min = \text{cap}(A, V \setminus A)$ denote the capacity of a mincut.
- Let $\text{cap}(v)$ be capacity of edges incident to vertex $v \in V_{n-i+1}$.
- Clearly, $\text{cap}(v) \geq \min$.
- Summing $\text{cap}(v)$ over all edges gives

$$2c(E) = 2\sum_{e \in E} c(e) = \sum_{v \in V} \text{cap}(v) \geq (n - i + 1) \cdot \min$$

- Hence, the probability of choosing an edge from the cut is at most $\min / c(E) \leq 2/(n - i + 1)$.

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- Hence, the probability of choosing an edge from the cut is at most $\text{min}/c(E) \leq 2/(n - i + 1)$.

Analysis

The probability that we do not choose an edge from the cut in iteration i is

$$1 - \frac{2}{n-i+1} = \frac{n-i-1}{n-i+1} \ .$$

The probability that the cut is alive after iteration $n-t$ (after which t nodes are left) is

$$\prod_{i=1}^{n-t} \frac{n-i-1}{n-i+1} = \frac{t(t-1)}{n(n-1)} \ .$$

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Repeating the algorithm $c\ln n\binom{n}{2}$ times gives that the probability that we are never successful is

$$\left(1-\frac{1}{\binom{n}{2}}\right)^{\binom{n}{2}c\ln n} \le \left(e^{-1/\binom{n}{2}}\right)^{\binom{n}{2}c\ln n} \le n^{-c} \enspace ,$$

where we used $1-x \le e^{-x}$.

Theorem 89

The randomized mincut algorithm computes an optimal cut with high probability. The total running time is $\mathcal{O}(n^4\log n)$.

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Improved Algorithm

Algorithm 53 RecursiveMincut($G = (V, E, c)$)

```
for $i = 1 \to n - n/\sqrt{2}$ do
    choose $e \in E$ randomly with probability $c(e)/C(E)$
    $G \leftarrow G/e$
if $|V| = 2$ return cut-value;
$cuta \leftarrow$ RecursiveMincut(G);
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- $T(n) = 2T(\frac{n}{\sqrt{2}}) + \mathcal{O}(n^2)$
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Improved Algorithm

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Algorithm 53 RecursiveMincut($G = (V, E, c)$)
for $i = 1 \to n - n/\sqrt{2}$ do
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Probability of Success

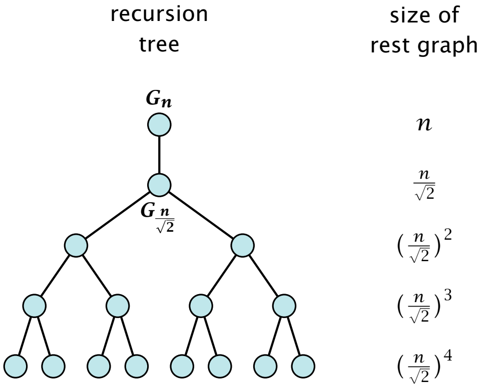
The probability of contracting an edge from the mincut during one iteration through the for-loop is only

$$\frac{t(t-1)}{n(n-1)} \approx \frac{t^2}{n^2} = \frac{1}{2} \ ,$$

as $t = \frac{n}{\sqrt{2}}$.

For the following analysis we ignore the slight error and assume that this probability is at most $\frac{1}{2}$.

Probability of Success



We can estimate the success probability by using the following game on the recursion tree. Delete every edge with probability $\frac{1}{2}$. If in the end you have a path from the root to at least one leaf node you are successful.

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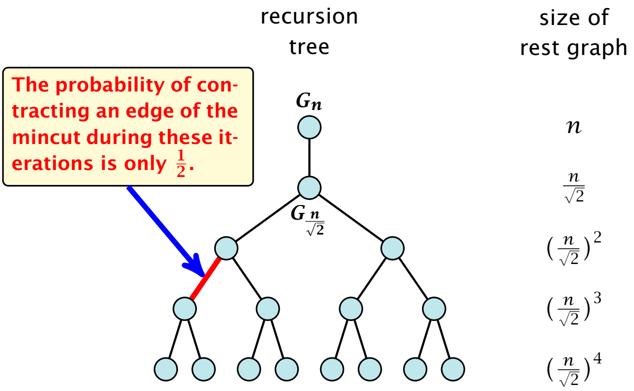

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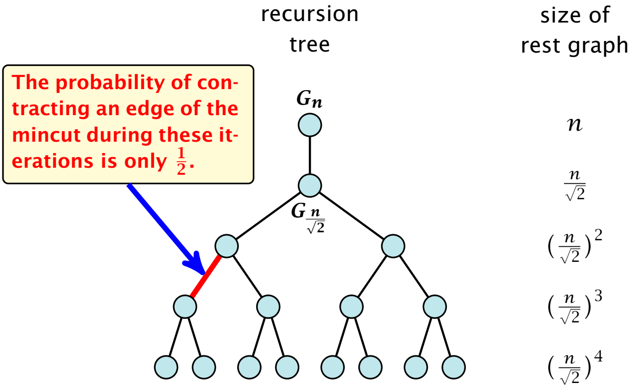

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Let for an edge e in the recursion tree, $h(e)$ denote the height (distance to leaf level) of the parent-node of e (end-point that is higher up in the tree). Let h denote the height of the root node.

Call an edge e alive if there exists a path from the parent-node of e to a descendant leaf, after we randomly deleted edges. Note that an edge can only be alive if it hasn't been deleted.

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16 Global Mincut

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One run of the algorithm can be performed in time $\mathcal{O}(n^2 \log n)$ *and has a success probability of* $\Omega(\frac{1}{\log n})$.

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