
Parallel Algorithms

Due Date: January 22, 2013 before class!

Problem 1 (10 Points)

Given an $r \times c$ array, show that the dimensions b_j and b_{j+2} for $1 \leq j \leq c - 3$ (and a_i and a_{i+2} for $1 \leq i \leq r - 3$, resp.) – as seen in Figure 3-7 on page 402 in Leighton’s book – are different.

Problem 2 (10 Points)

(a) Show that there are $\binom{L-1}{L_0-1}$ ways to choose the length l_1, l_2, \dots, l_{L_0} of the stagnant paths in the proof of Theorem 3.4 in Leighton’s book (the Theorem about the flip-bit algorithm) so that

$$l_1 + l_2 + \dots + l_{L_0} = L$$

and $l_i \geq 1$ for $1 \leq i \leq L_0$.

Hint: Show that there is a bijective mapping between $\{l_1, l_2, \dots, l_{L_0}\}$ and $(L - 1)$ -bit binary strings with $L_0 - 1$ 1s.

(b) Show that

$$\binom{L-1}{L_0-1} \leq \binom{L}{L_0}$$

for any $1 \leq L_0 \leq L$.

Problem 3 (10 Points)

Show that an inorder labeling of an $(N - 1)$ -node complete binary tree induces an embedding in the N -node hypercube with dilation 2.

Problem 4 (10 Points)

Regarding the flip-bit algorithm, explain in detail why there is at most one way to choose the flip bits in nodes of T that are contained in traces.