## Lemma 12 (Chernoff Bounds)

Let $X_1, \ldots, X_n$ be $n$ *independent* 0-1 random variables, not necessarily identically distributed. Then for $X = \sum_{i=1}^{n} X_i$ and $\mu = E[X]$, $L \leq \mu \leq U$, and $\delta > 0$

$$\Pr[X \geq (1+\delta)U] < \left( \frac{e^{\delta}}{(1+\delta)^{1+\delta}} \right)^{U} \; ,$$

*and*

$$\Pr[X \leq (1-\delta)L] < \left( \frac{e^{-\delta}}{(1-\delta)^{1-\delta}} \right)^{L} \; ,$$

**Lemma 13**

*For $0 \leq \delta \leq 1$ we have that*

$$\left( \frac{e^{\delta}}{(1+\delta)^{1+\delta}} \right)^{U} \leq e^{-U\delta^2/3}$$

*and*

$$\left( \frac{e^{-\delta}}{(1-\delta)^{1-\delta}} \right)^{L} \leq e^{-L\delta^2/2}$$

# Integer Multicommodity Flows

▶ Given $s_i$-$t_i$ pairs in a graph.

▶ Connect each pair by a paths such that not too many path use any given edge.

$$
\begin{array}{rlcl}
\min & & W & \\
\text{s.t.} \quad \forall i & \sum_{p \in \mathcal{P}_i} x_p & = & 1 \\
& \sum_{p:e \in p} x_p & \leq & W \\
& x_p & \in & \{0,1\}
\end{array}
$$

# Integer Multicommodity Flows

**Randomized Rounding:**

For each $i$ choose one path from the set $\mathcal{P}_i$ at random according to the probability distribution given by the Linear Programming Solution.

**Theorem 14**

*If $W^* \geq c \ln n$ for some constant $c$, then with probability at least $n^{-c/3}$ the total number of paths using any edge is at most $W^* + \sqrt{cW^* \ln n}$.*

Let $X_e^i$ be a random variable that indicates whether the path for $s_i$-$t_i$ uses edge $e$.

Then the number of paths using edge $e$ is $Y_e = \sum_i X_e^i$.

Let $X_e^i$ be a random variable that indicates whether the path for $s_i$-$t_i$ uses edge $e$.

Then the number of paths using edge $e$ is $Y_e = \sum_i X_e^i$.

Let $X_e^i$ be a random variable that indicates whether the path for $s_i$-$t_i$ uses edge $e$.

Then the number of paths using edge $e$ is $Y_e = \sum_i X_e^i$.

# Integer Multicommodity Flows

Let $X_e^i$ be a random variable that indicates whether the path for $s_i$-$t_i$ uses edge $e$.

Then the number of paths using edge $e$ is $Y_e = \sum_i X_e^i$.

$$E[Y_e] = \sum_i \sum_{p \in P_i : e \in p} x_p^* = \sum_{p : e \in P} x_p^* \leq W^*$$

# Integer Multicommodity Flows

Let $X_e^i$ be a random variable that indicates whether the path for $s_i$-$t_i$ uses edge $e$.

Then the number of paths using edge $e$ is $Y_e = \sum_i X_e^i$.

$$E[Y_e] = \sum_i \sum_{p \in \mathcal{P}_i : e \in p} x_p^* = \sum_{p : e \in P} x_p^* \leq W^*$$

# Integer Multicommodity Flows

Let $X_e^i$ be a random variable that indicates whether the path for $s_i$-$t_i$ uses edge $e$.

Then the number of paths using edge $e$ is $Y_e = \sum_i X_e^i$.

$$E[Y_e] = \sum_i \sum_{p \in \mathcal{P}_i : e \in p} x_p^* = \sum_{p : e \in P} x_p^* \leq W^*$$

# Integer Multicommodity Flows

Choose $\delta = \sqrt{(c \ln n)/W^*}$.

Then

$$\Pr[Y_e \geq (1 + \delta)W^*] < e^{-W^* \delta^2/3} = \frac{1}{n^{c/3}}$$

# Integer Multicommodity Flows

Choose $\delta = \sqrt{(c \ln n)/W^*}$.

Then
$$\Pr[Y_e \geq (1 + \delta)W^*] < e^{-W^* \delta^2/3} = \frac{1}{n^{c/3}}$$