# Project Selection

**Project selection problem:**

- Set $P$ of possible projects. Project $v$ has an associated profit $p_v$ (can be positive or negative).
- Some projects have requirements (taking course EA2 requires course EA1).
- Dependencies are modelled in a graph. Edge $(u, v)$ means "can't do project $u$ without also doing project $v$."
- A subset $A$ of projects is feasible if the prerequisites of every project in $A$ also belong to $A$.

**Goal:** Find a feasible set of projects that maximizes the profit.

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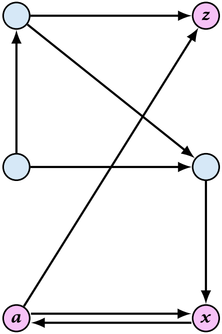
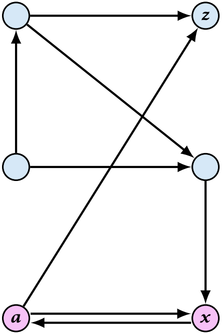
**The prerequisite graph:**

- $\{x, a, z\}$ is a feasible subset.
- $\{x, a\}$ is infeasible.

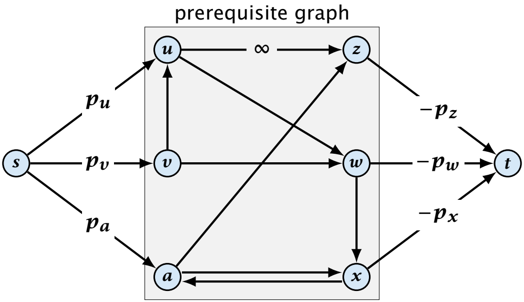


# Project Selection

**Mincut formulation:**

- Edges in the prerequisite graph get infinite capacity.
- Add edge $(s,v)$ with capacity $p_v$ for nodes $v$ with positive profit.
- Create edge $(v,t)$ with capacity $-p_v$ for nodes $v$ with negative profit.



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*$A$ is a mincut if $A \setminus \{s\}$ is the optimal set of projects.*

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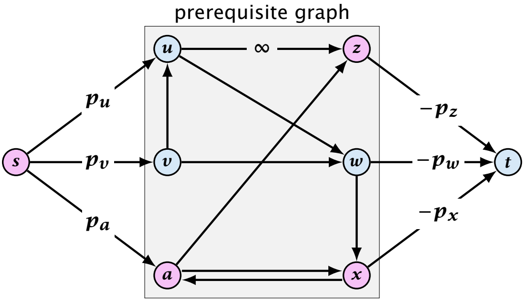
**Proof.**

- $A$ is feasible because of capacity infinity edges.



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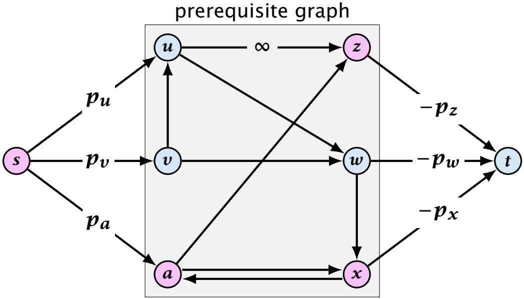

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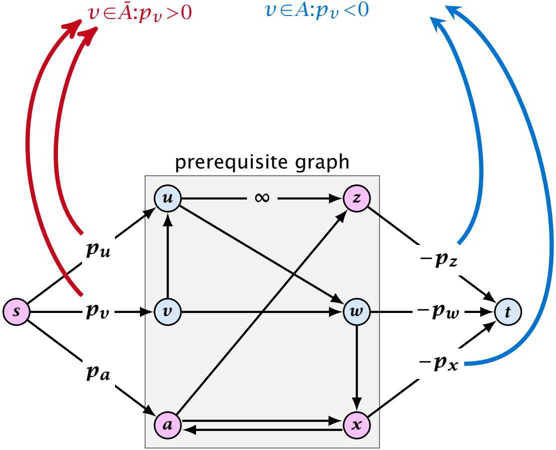

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$$= \sum_{v: p_v > 0} p_v - \sum_{v \in A} p_v$$