Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Wintersemester 2013
Assignment 1
21 October 2013

Efficient Algorithms and Datastructures I

Question 1 (10 Points)

Prove the following statements:

1. $f(n) + g(n) \in \Omega(f(n))$
2. $f(n) \in O(g(n)) \Rightarrow f(n) + g(n) \in O(g(n))$
3. $f(n) \in o(g(n))$ and $g(n) \in O(h(n)) \Rightarrow h(n) \in \omega(f(n))$
4. $f(n) \in O(g(n))$ and $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$

State whether the following statement is true:

1. $\frac{1}{\Omega(n)} \subseteq O(\frac{1}{n})$

Question 2 (10 Points)

For constants $c, \epsilon > 0$ and $k > 1$, arrange the following functions of n in non-decreasing asymptotic order so that $f_i(n) = O(f_{i+1}(n))$ for two consecutive functions in your sequence. Also indicate whether $f_i(n) = \Theta(f_{i+1}(n))$ holds or not.

$$n^k, \sqrt{n}, 2^n, n^{1+sin(n)}, \log(n!), n^{k+\epsilon}, n^n, n, n^k(\log n)^c, n!, n\log n, 3^n, n\log\log n, n\log(n^2)$$

Question 3 (5 Points)

Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of Θ-notation, prove that $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.

Question 4 (5 Points)

Show that for any real constants a and b, where $b > 0$,

$$(n + a)^b = \Theta(n^b)$$