Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Winter Term 2013
Assignment 3
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Efficient Algorithms and Data Structures I

Question 1 (10 Points)

Solve the following recurrence relations:

1. $a_n = -a_{n-1} + 9a_{n-2} - 11a_{n-3} + 4a_{n-4}$ with $a_0 = -7$, $a_1 = 4$, $a_2 = 48$ and $a_3 = 0$.

Question 2 (10 Points)

Calculate the value of $\sum_{i=1}^{n} i^2$ by setting up a recurrence relation; transforming it into a homogeneous relation via the method developed in the lecture and then solving this relation via the characteristic polynomial.

Question 3 (10 Points)

Give tight asymptotic bounds for the following recurrence relation:

$$T(n) = T(\sqrt{n}) + 1$$

Extra Question 4 (10 Points)

Give tight asymptotic bounds for the following recurrence relation:

$$T(n) = T(\frac{n}{\log n}) + 1$$

Hint: How often do you have to apply the iteration $n \mapsto n/\log n$ until the problem size drops to $\sqrt{n}$? How often do you have to apply it to bring it down from $\sqrt{n}$ to $\sqrt{\sqrt{n}}$? Also use the fact that $\sum_{i=1}^{k} \frac{2^i}{i} = \mathcal{O}(\frac{2^k}{k})$.

Question 5 (10 Points)

Solve the following recurrence relations using generating functions:

1. $a_n = a_{n-1} + 2^{n-1}$ for $n \geq 1$ with $a_0 = 2$.
2. $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$ for $n \geq 3$ with $a_0 = a_1 = a_2 = 1$.