Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Wintersemester 2013
Assignment 9
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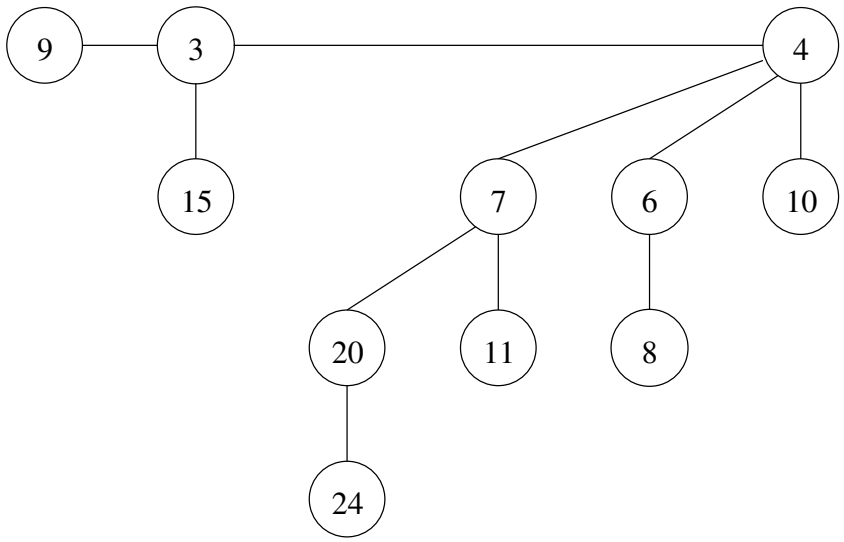
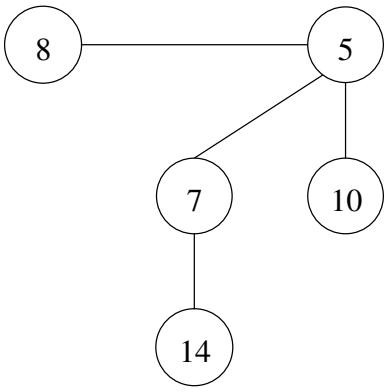
Efficient Algorithms and Datastructures I

Question 1 (10 Points)

Consider the following Binomial Heaps:
Heap A:



Heap B:



Carry out the following operations sequentially on the heaps and show them after each operation(always carry out each operation on the result of the previous operation):

1. merge(A,B)
2. deleteMin()

Question 2 (10 Points)

We say that $f(n) = \overset{\infty}{\Omega}(g(n))$ if there exists a positive constant c such that $f(n) \geq cg(n) \geq 0$ for infinitely many integers n. Find inputs that cause DELETE-MIN, DECREASE-KEY, and DELETE to run in $\Omega(\log n)$ time for a binomial heap. Explain why the worst-case running times of INSERT, MINIMUM, and MERGE are $\overset{\infty}{\Omega}(\log n)$ but not $\Omega(\log n)$ for a binomial heap.

Question 3 (10 Points)

n motorcyclists $M_1, M_2, \ldots, M_n$ start riding their bikes from a (straight) start line. At the start M_i and M_{i+1} are adjacent to each other. Each motorcyclist M_i starts at some angle ϕ_i and keeps riding in a straight line along this direction at a constant speed $s_i > 0$. Whenever a motorcyclist M_j comes across the path traversed by any other motorcyclist M_i, we say that M_i defeated M_j and in that case, M_j stops riding.

(a) We call the point where M_i defeats M_j as the point of ambush $A_{i,j} \in \mathbb{R}^2$. Show that if $A_{i',j'}$ is a point of ambush which occurs closest to the start line, then i' and j' are consecutive integers.

(b) Show how to enumerate in $O(n \log n)$ time, all events where one motorcyclist defeats another.

Question 4 (10 Points)

Give a sequence of m MAKESET, UNION and FIND operations, n of which are MAKESET operations, that take $\Omega(m \log n)$ time when we use union by rank only.