Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Winter Semester 2013/14
Problem Set 7
December 3rd, 2013

Parallel Algorithms

Due date: December 10th, 2013 **before** *class!*

Problem 1 (10 Points)

Define a bisection as a cut of a graph, i.e. a subset of nodes or edges, such that the graph is partitioned into two equally sized parts.
Given an d-dimensional hypercube, show that the removal of the nodes with size $\lceil \frac{d}{2} \rceil$ and size $\lfloor \frac{d}{2} \rfloor$ (i.e. nodes with that many 1s) results in a bisection containing $\Theta\left(\frac{2^d}{\sqrt{d}}\right)$ nodes.

Problem 2 (10 Points)

Let u and v be nodes of the d-dimensional hypercube, and let $u_1, u_2, \ldots, u_d$ and $v_1, v_2, \ldots, v_d$ denote their neighbors, respectively. Let π be any permutation on $\{1, 2, \ldots, d\}$. Show that there is an automorphism of the hypercube σ such that $\sigma(u) = v$ and $\sigma(u_i) = v_{\pi(i)}$ for $1 \leq i \leq d$.
Hint: An *automorphism* of a graph is a one-to-one mapping of the nodes to the nodes such that edges are mapped to edges.

Problem 3 (10 Points)

Prove that the d-dimensional wrapped butterfly is Hamiltonian for $d \geq 2$.
Hint: You may try an induction on the dimension d of the network.