Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Winter Semester 2014/15
Problem set 3
October 21, 2014

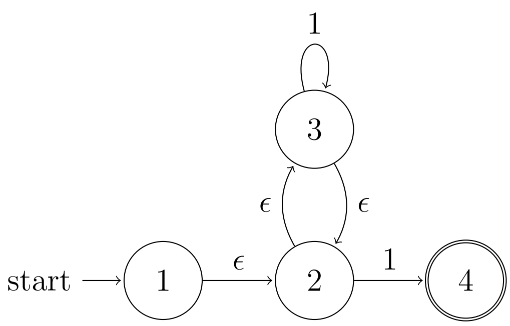
Automata and Formal Languages

Due October 28, 2014 before class!

Exercise 1 (Conversions - 10 points)

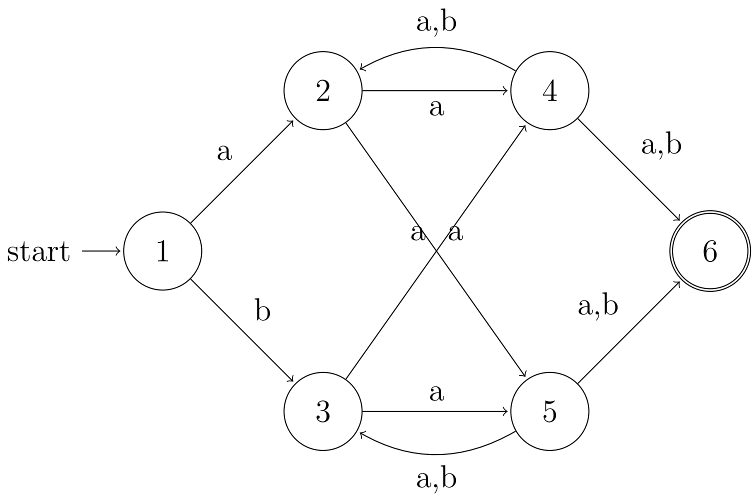
(a) Let $r = (0+1)^*1((01)^*+1)^*$ be a regular expression over the alphabet $\Sigma = \{0, 1\}$. Use the rules from the lecture to transform the regular expression into an ϵ-NFA. Give intermediate steps and specify which rules you use.

(b) Convert the following ϵ-NFA into an NFA using the procedure from the lecture.



Exercise 2 (Minimization - 10 points)

Consider the following NFA A:



(a) Convert A into a DFA.

(b) Minimize the resulting DFA using one of the methods from the lecture. Give intermediate steps. Check your results using JFLAP.

Exercise 3 (Minimization II - 10 points)

(a) In the lecture we saw that the problem of NFA minimization is PSPACE-complete, therefore instead of a method for minimizing an NFA, we looked at a method for reducing the size of an NFA.
Give an example of an NFA different from the one in class, that is not minimal after an application of the reduction algorithm from the lecture.

(b) Prove or disprove:
For every regular language L, the minimal DFA recognizing L has the same number of states as the minimal DFA recognizing the complement of L.

Exercise 4 (Star-free expressions - 10 points)

The set $\mathbb{S}_\Sigma$ of *star-free expressions* is inductively defined as

$$\begin{aligned} S^0 &= \Sigma \cup \{\epsilon, \emptyset\} \\ S^{k+1} &= \{(\phi + \psi), (\phi\psi), \overline{\phi}, (\phi \cap \psi) \mid \phi, \psi \in S^k\} \\ \mathbb{S}_\Sigma &= \bigcup_{k \in \mathbb{N}} S^k \end{aligned}$$

The language $\mathcal{L}(\rho)$ induced by a star-free expression $\rho \in \mathbb{S}_\Sigma$ is defined similarly to regular expressions:

$$\mathcal{L}(\rho) := \begin{cases} \emptyset & \text{if } \rho = \emptyset & \mathcal{L}(\phi) \cup \mathcal{L}(\psi) & \text{if } \rho = (\phi + \psi) \\ \{\rho\} & \text{if } \rho \in \Sigma \cup \{\epsilon\} & \mathcal{L}(\phi) \cdot \mathcal{L}(\psi) & \text{if } \rho = (\phi \cdot \psi) \\ \Sigma^* \setminus \mathcal{L}(\phi) & \text{if } \rho = \overline{\phi} & \mathcal{L}(\phi) \cap \mathcal{L}(\psi) & \text{if } \rho = (\phi \cap \psi). \end{cases}$$

(a) Prove that the regular expression a^* describes a star-free language.

(b) Prove that the regular expression $(ab)^*$ describes a star-free language.

(c) Show that for every star-free expression $\phi \in \mathbb{S}_\Sigma$, $\mathcal{L}(\phi)$ is a regular language.