
Automata and Formal Languages

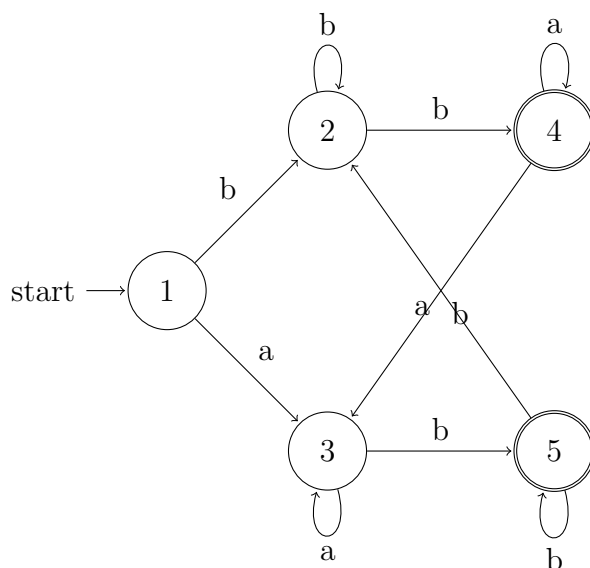
Due January 13, 2015 before class!

Exercise 1 (Semilinear sets - 10 points)

A set of integers is linear if it is of the form $\{c + pi \mid i = 0, 1, 2, \dots\}$ for some constants c and p . A set is semilinear if it is the finite union of linear sets. Let $R \subseteq 0^*$ be regular. Prove that $\{i \mid 0^i \in R\}$ is semi-linear.

Exercise 2 (Büchi-automata I - 10 points)

Consider the following NBA:



Find a number $n \geq 1$ and regular languages $U_1, V_1, \dots, U_n, V_n$ s.t.

$$L = \bigcup_{i=1}^n U_i V_i^\omega$$

Exercise 3 (Büchi Automata II - 10 points)

Find a language over the alphabet $\Sigma = \{a, b\}$ consisting of one infinite word such that there is no Büchi-automaton recognizing it.

Exercise 4 (Büchi Automata III - 10 points)

Let $\text{inf}(w)$ denote the set of all letters $a \in \Sigma$ that occur infinitely often in w .

Construct a Büchi automaton over the alphabet $\Sigma = \{a, b, c\}$ that recognizes the language L where

(a) $L = \{w \mid \{a, b\} \subseteq \text{inf}(w)\}$

(b) $L = \{w \mid \{a, b\} = \text{inf}(w)\}$

(c) $L = \{w \in \{a, b, c\}^\omega \mid \text{if } a \in \text{inf}(w) \text{ then } \{b, c\} \subseteq \text{inf}(w)\}$