

## Baseball Elimination

### Proof ( $\Rightarrow$ )

- ▶ Suppose we have a flow that saturates all source edges.
- ▶ We can assume that this flow is **integral**.
- ▶ For every pairing  $x$ - $y$  it defines how many games team  $x$  and team  $y$  should win.
- ▶ The flow leaving the team-node  $x$  can be interpreted as the additional number of wins that team  $x$  will obtain.
- ▶ This is less than  $M - w_x$  because of capacity constraints.
- ▶ Hence, we found a set of results for the remaining games, such that no team obtains more than  $M$  wins in total.
- ▶ Hence, team  $z$  is not eliminated.

## Project Selection

### Project selection problem:

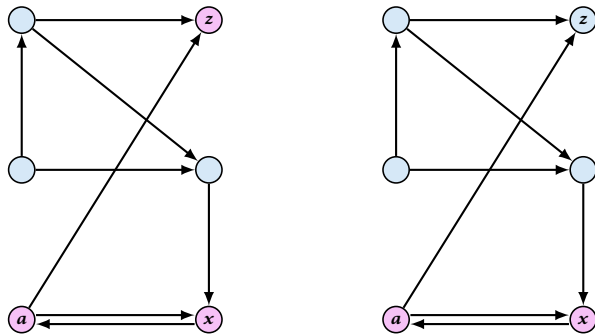
- ▶ Set  $P$  of possible projects. Project  $v$  has an associated profit  $p_v$  (can be positive or negative).
- ▶ Some projects have requirements (taking course EA2 requires course EA1).
- ▶ Dependencies are modelled in a graph. Edge  $(u, v)$  means “can’t do project  $u$  without also doing project  $v$ .”
- ▶ A subset  $A$  of projects is **feasible** if the prerequisites of every project in  $A$  also belong to  $A$ .

**Goal:** Find a feasible set of projects that maximizes the profit.

## Project Selection

### The prerequisite graph:

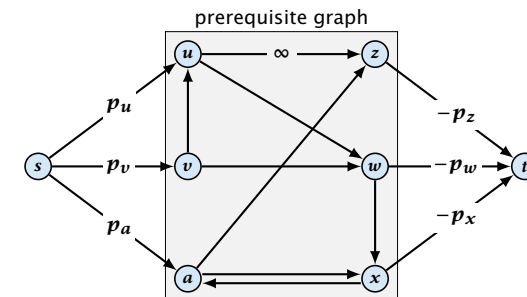
- ▶  $\{x, a, z\}$  is a feasible subset.
- ▶  $\{x, a\}$  is infeasible.



## Project Selection

### Mincut formulation:

- ▶ Edges in the prerequisite graph get infinite capacity.
- ▶ Add edge  $(s, v)$  with capacity  $p_v$  for nodes  $v$  with positive profit.
- ▶ Create edge  $(v, t)$  with capacity  $-p_v$  for nodes  $v$  with negative profit.



## Theorem 2

$A$  is a mincut if  $A \setminus \{s\}$  is the optimal set of projects.

**Proof.**

▶  $A$  is feasible because of capacity infinity edges.

$$\text{cap}(A, V \setminus A) = \sum_{v \in \bar{A}; p_v > 0} p_v + \sum_{v \in A; p_v < 0} (-p_v)$$

$$= \sum_{v: p_v > 0} p_v - \sum_{v \in A} p_v$$

For the formula we define  $p_s := 0$ .

The step follows by adding  $\sum_{v \in A; p_v > 0} p_v - \sum_{v \in A; p_v > 0} p_v = 0$ .

Note that minimizing the capacity of the cut  $(A, V \setminus A)$  corresponds to maximizing profits of projects in  $A$ .

