Technische Universität München
Fakultät für Informatik
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Problem Set 10
December 15th, 2014

Parallel Algorithms

Due date: January 12th, 2015 **before** *class!*

Problem 1 (10 Points)

Prove that the d-dimensional wrapped butterfly is Hamiltonian for $d \geq 2$.
Hint: You may try an induction on the dimension d of the network.

Problem 2 (10 Points)

An *embedding* of a network A into another network B is a mapping of the nodes of A onto the nodes of B and a mapping of the edges of A onto paths in B such that for all pairs of connected nodes in A, it holds that the pair of corresponding mapped nodes is also connected in B. The *dilation* of an embedding is the maximum amount that any edge of A must be "stretched" in B, where stretching an edge of A means that it may be mapped across more than one edge of B.
Show how the CCC(d) is contained in the BF(d) with wrap-around edges, in particular, that it can be embedded with a dilation of only 2.

Problem 3 (10 Points)

Define the *greedy routing* algorithm on a butterfly as: Every packet crosses the hypercube dimensions in increasing order.
In addition, define the *node congestion* to be the highest number of path crossings in any node in the graph. (For node-disjoint routing, the congestion is 1.)
Consider the following two routing problems:

1. A *bit-reversal* permutation maps $x_1x_2\dots x_d$ to $x_dx_{d-1}\dots x_1$.
2. A *transpose* permutation maps $x_1x_2\dots x_d$ to $x_{d/2+1}\dots x_dx_1\dots x_{d/2}$.

Assume that the input bits are given in the nodes on level 0, and that they should be routed to the nodes on level $d-1$.
Show that the greedy routing algorithm must have a node congestion of $\Omega(\sqrt{n})$ for these two permutation routing problems.