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Problem set 0
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Online and Approximation Algorithms

Exercise 1 (Ski Rental)

The ski rental problem is defined as follows: Assume that renting a pair of skis costs 1 per day while buying a pair of skis costs b. Every day we have to decide, in an online fashion whether we want to continue renting skis for another day or buy a pair of skis. At some unknown time D, we will break our leg and have to quit skiing. Our goal is to minimize the cost of skiing.

(a) What is the optimal offline cost?

(b) Develop a strongly $(2 - \frac{1}{b})$-competitive online algorithm ALG for the ski rental problem and prove its competitiveness (a strongly c-competitive algorithm ALG satisfies $\text{ALG}(\sigma) \leq c \cdot \text{OPT}(\sigma)$ for all request sequences σ).

Exercise 2 (Growing the fast memory)

Recall that FIFO is the online paging algorithm that evicts the page that has been in the fast memory for the longest time, LRU is the online paging algorithm that evicts the page that has been used least recently and OPT is the optimal offline algorithm.
Consider the following request sequence

$$\sigma = \text{ABCDABEABCDE} .$$

Assume that we start with an empty fast memory. Show the content of the fast memory during executions of OPT, LRU and FIFO on σ for fast memory sizes 3 and 4. Count the number of page faults during the executions. What do you notice?

Exercise 3 (FIFO)

Recall that FIFO is the online paging algorithm that evicts the page that has been in the fast memory for the longest time. Build a sequence on which the competitive ratio is equal to k, where k is the number of pages that fit in fast memory.