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## Online and Approximation Algorithms

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*Due June 29, 2015 before class!*

### **Exercise 1 (Metrical Task Systems - 10 points)**

Show that the ski rental problem and the  $k$ -server problem can be formulated as metrical task system problems.

### **Exercise 2 (Next-Fit Bin Packing - 10 points)**

In the bin packing problem, we are given a set of items  $I = \{1, 2, \dots, n\}$  which have to be packed into bins. Each item  $i \in I$  has a volume  $0 \leq v_i \leq 1$  and it has to be assigned entirely to a single bin. The capacity of a bin is equal to 1. We would like to pack the items in a way that the total volume of the items stored in a bin does not exceed its capacity. Our objective is to minimize the number of used bins. Algorithm *Next-Fit* takes an arbitrary order of the items and it packs them one by one. It maintains an *open bin* and, for each item  $i \in I$ , if  $i$  fits in the open bin, then Next-Fit puts it in. Otherwise, the bin is closed and a new bin is opened. Show that Next-Fit is a 2-approximation. Next, show that we cannot expect a better approximation ratio for it.

### **Exercise 3 (Eulerian Cycle - 10 points)**

Show that a connected graph  $G = (V, E)$ , with possibly multiple edges between a pair of vertices, contains an Eulerian cycle if and only if every vertex  $v \in V$  is of even degree.

### **Exercise 4 (1-2 TSP on Complete Graphs - 10 points)**

The 1-2 TSP is the special case of TSP in which the length of every edge of the input graph  $G$  is either 1 or 2. Consider the algorithm which starts from an arbitrary tour of a complete graph  $G$  and improves the tour iteratively until no further improvement is possible. A tour  $C = \{v_1, v_2, \dots, v_n, v_1\}$  may be improved follows. Four different vertices  $v_i, v_{i+1}, v_j, v_{j+1}$  of  $C$  are picked and the edges  $(v_i, v_{i+1}), (v_j, v_{j+1})$  are replaced by the edges  $(v_i, v_j), (v_{i+1}, v_{j+1})$  so as to obtain a new shorter cycle  $C'$ . Show that this algorithm is a  $\frac{3}{2}$ -approximation for the 1-2 TSP.