### **Baseball Elimination**

#### Proof (⇒)

- ▶ Suppose we have a flow that saturates all source edges.
- ▶ We can assume that this flow is integral.
- ightharpoonup For every pairing x-y it defines how many games team xand team  $\gamma$  should win.
- ightharpoonup The flow leaving the team-node x can be interpreted as the additional number of wins that team x will obtain.
- ▶ This is less than  $M w_x$  because of capacity constraints.
- ▶ Hence, we found a set of results for the remaining games, such that no team obtains more than *M* wins in total.
- ▶ Hence, team z is not eliminated.

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12.2 Baseball Elimination

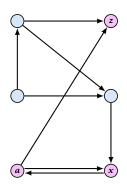
442

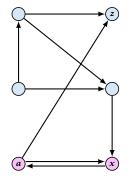
444

# **Project Selection**

## The prerequisite graph:

- $\rightarrow$  {x, a, z} is a feasible subset.
- $\blacktriangleright$  {x, a} is infeasible.





# **Project Selection**

#### **Project selection problem:**

- ▶ Set *P* of possible projects. Project *v* has an associated profit  $p_{\nu}$  (can be positive or negative).
- Some projects have requirements (taking course EA2 requires course EA1).
- ▶ Dependencies are modelled in a graph. Edge (u, v) means "can't do project u without also doing project v."
- ► A subset A of projects is feasible if the prerequisites of every project in A also belong to A.

**Goal:** Find a feasible set of projects that maximizes the profit.

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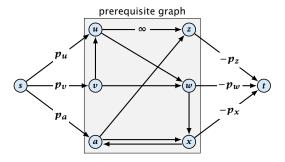
12.3 Project Selection

443

# **Project Selection**

#### Mincut formulation:

- Edges in the prerequisite graph get infinite capacity.
- Add edge (s, v) with capacity  $p_v$  for nodes v with positive profit.
- Create edge (v,t) with capacity  $-p_v$  for nodes v with negative profit.



12.3 Project Selection

# **Theorem 2** A is a mincut if $A \setminus \{s\}$ is the optimal set of projects. Proof. ► *A* is feasible because of capacity infinity edges. $cap(A, V \setminus A) = \sum_{v \in \overline{A}: p_v > 0} p_v + \sum_{v \in A: p_v < 0} (-p_v)$ $= \sum_{v: p_v > 0} p_v - \sum_{v \in A} p_v$ prerequisite graph For the formula we define $p_s := 0$ . The step follows by adding $\sum_{v \in A: p_v > 0} p_v - \sum_{v \in A: p_v > 0} p_v$ $\sum_{v \in A: p_v > 0} p_v = 0.$ Note that minimizing the capacity of the cut $(A, V \setminus A)$ corresponds to maximizing profits of projects in A.

